

P L A I N
TRIGONOMETRY

RENDERED EASY AND FAMILIAR,

BY CALCULATIONS IN ARITHMETICK ONLY:

WITH ITS

APPLICATION AND USE

IN ASCERTAINING

All Kinds of HEIGHTS, DEPTHS, and DISTANCES,

IN

The HEAVENS, as well as on the EARTH and SEAS;

WHETHER OF

TOWERS, FORTS, TREES, PYRAMIDS, COLUMNS, WELLS, SHIPS, HILLS,
CLOUDS, THUNDER AND LIGHTNING, ATMOSPHERE, SUN, MOON,
PLANETS, MOUNTAINS IN THE MOON, SHADOWS OF EARTH AND
MOON, BEGINNING AND END OF ECLIPSES, &c.

IN WHICH IS ALSO SHEWN,

A CURIOUS TRIGONOMETRICAL METHOD of discovering the Places where BEES HIVE in
LARGE WOODS, in Order to obtain, more readily, the SALUTARY PRODUCE of those
LITTLE INSECTS.

By the Rev. R. TURNER, of *Magdalen Hall, Oxford*;

Author of *The View of the Earth*; *View of the Heavens*;—*System of Gauging*;—and
Chronologer Perpetual :

Rector of COMBERTON, Vicar of ELMLEY, Minister of STOULTON, and Chaplain to
the Right Honourable the COUNTESS DOWAGER of WIGTON.

*Cuncta Trigonus habet, satagitquædocta Mathesis,
Ille aperit clausum, quicquid Olympus habet.*

*Within the grand Triangle lies unveil'd,
What Sages sought for, and what Heaven conceal'd.*

L O N D O N :

PRINTED FOR S. CROWDER, WATER-NOSTER ROW. 1778.

TO THOSE
GENTLEMEN,

WHOSE GENIUS MAY INCLINE, OR EMPLOYMENT
LEAD THEM TO THE STUDY OF THE

MATHEMATICKS.

GENTLEMEN,

TRIGONOMETRY has always been look'd on as one of the most useful Branches of *Mathematical* Learning. *Navigation, Surveying, Astronomy, &c.* stand wholly upon this *Basis*. But the common Method of answering these Problems by large TABLES of *Sines, Tangents, and Secants*, renders it not only expensive by the Purchase of them; but often precarious in the Solution, through Mistakes of the Press. I have therefore, for the Use of the *Young Mathematician*, (from a Consideration of what has been published on this curious Subject) compos'd the present *System*, by which any of the *Cases* in *Right* or *Oblique Plain Triangles* may be answered on the *Spot*, by an easy Calculation in *Arithmetick* only *. The great Advantages resulting from this Method to *Gentlemen* in the *Army* or *Navy*, as well as to Those in their *private Studies* at *Home*, must immediately appear; as it will be found to answer the most necessary Problems as *expeditiously* as *Logarithms*, (oftentimes more so;) and at the same Time wholly deliver you from those *voluminous Tables* and the *inartificial Fatigues* of carrying them always with you.—Should this little Treatise be so happy as to meet your Approbation, it will give a particular Pleasure to,

Your most humble Servant,



The AUTHOR.

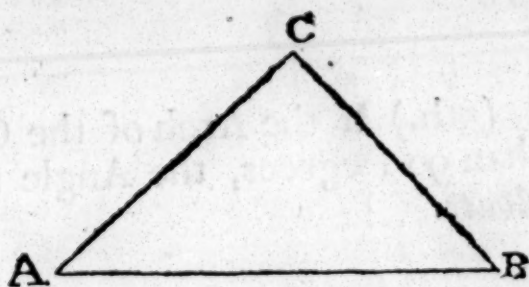
* It is wished that some Gentleman would undertake the Publication of *Spherical Trigonometry* in a similar Manner.



P L A I N TRIGONOMETRY.

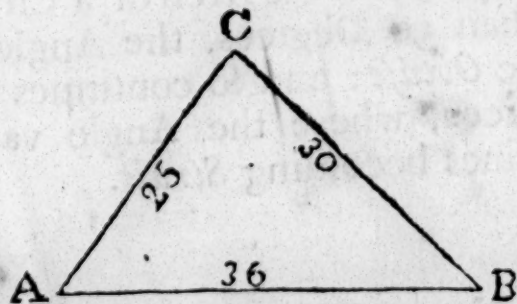
TRIGONOMETRY is that Part of *Mathematicks*, which is employed in calculating the *Sides* and finding the *Angles* of any *Triangle* required; it is of the greatest *Use*, as nothing in *Navigation*, *Astronomy*, &c. can be done without it; and depends on the Knowledge of the following Observations, or Properties of that Figure.

(1st.) Every Triangle consists of Six Parts; that is, of *Three Sides* and *Three Angles*, as in the Figure ABC; the Three Sides are, AB, AC, CB, and the Three Angles, A, B, C.



NOTE. Sometimes an *Angle* is expressed by *Three Letters*; in that Case, the *Middle Letter* denotes the *Angular Point*. Thus, ABC expresses the Angle B; BAC the Angle A; and ACB the Angle C.

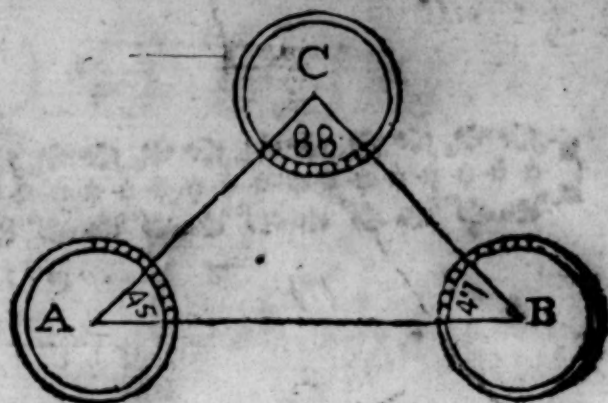
(2d.) The *Sides* of all plain Triangles are measured by a Line of equal *Parts*, as of *Inches*,—*Feet*,—*Yards*,—or *Leagues*.



Thus, The Side AB is 36 Leagues.—The Side AC 25 Leagues.—And the Side BC is 30 Leagues.

(3d.) The

(3d.) The *Angles* are measured by the *Arch* of a *Circle* described upon the *Angular Point*, and contained between the *Two Legs* that form the *Angle*.

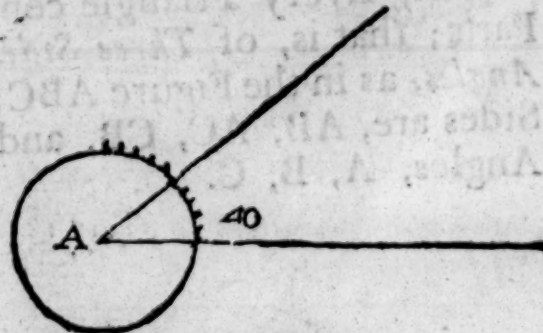


NOTE. Every Circle is divided into 360 equal Parts, called *Degrees*; each of which is divided into 60 more, called *Minutes*: And the Number of Degrees contained between the Two Legs, that constitute the Angle, is the *Measure* of that Angle. Thus, The Angle A is 45 *Degrees*.---The Angle B 47.---The Angle C 88.

(4th.) If the Arch of a Circle intercepted between the Two Legs be exactly 90 Degrees, the Angle is called a *Right Angle*, and the Legs are *perpendicular* to one another.

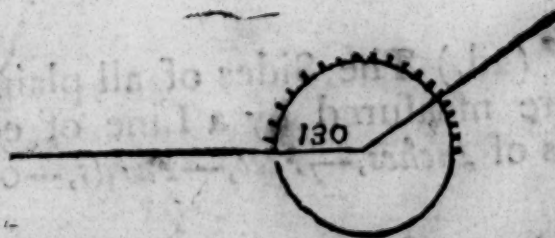


(5th.) If the Arch of the Circle be *less* than 90 Degrees, the Angle is said to be *Acute*.



NOTE. What an *Acute* Angle wants of 90 Degrees, is called the *Complement* of that Angle. Thus, Suppose the Angle A was 40 Degrees; then its Complement is 50 Degrees; for 40 added to 50 make 90, as observed before.

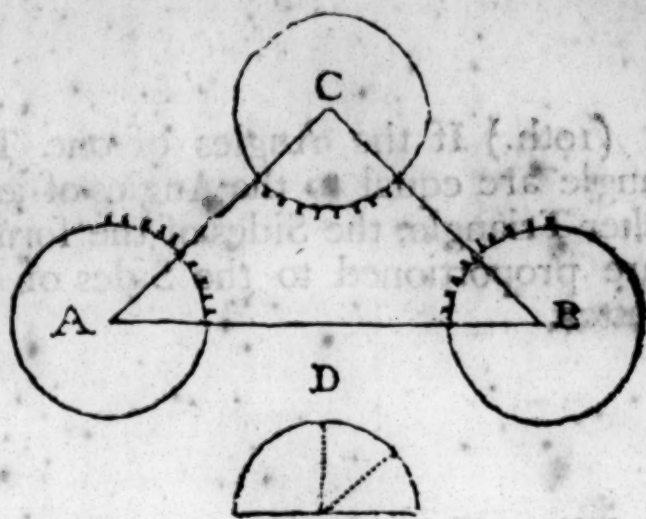
(6th.) If the Arch of a Circle be *more* than 90 Degrees, the Angle is said to be *Obtuse*; and so continues to 180 Degrees, where the Angle vanishes, the Lines becoming *Strait*.



NOTE. What an *Obtuse* Angle wants of 180 Degrees, is also called the *Complement* of that Angle to a *Semicircle*.

(7th.) The

(7th.) The Three Angles of every plain Triangle, being taken together, make 180 Degrees (equal to a Semicircle), and this they always do, let the Triangle be drawn however you please.

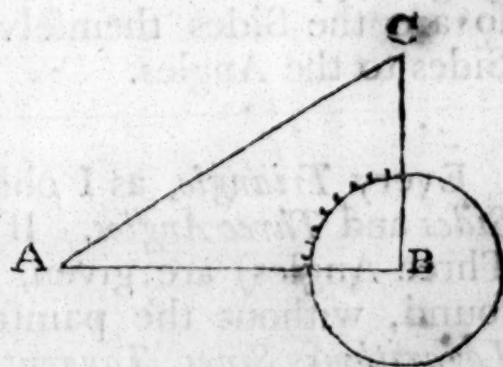


Thus, If the *Semicircle D* be drawn with the same *Radius*, or opening of the Dividers, as the little Circles on the Angles A, B, C, are; you will find, by taking off the several Arches, and applying them to the Semicircle, that they will just fill it up, and thereby make 180 Degrees; because every Semicircle contains that Number of Degrees.

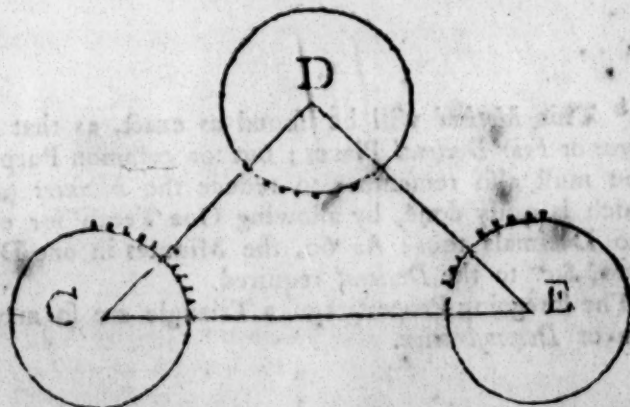
Hence it is evident, that if one Angle be a *Right One*, the other Two will be *Acute*; and taken together, be equal to one *Right Angle*, or just 90 Degrees.

Hence also, if *Two* Angles of any Triangle are known, the *Third* is easily found, being only the *Degrees* the other Two Angles want of 180.

(8th.) If a *Triangle* has one *Right Angle*, it is called a *Right Angled Triangle*: Thus ABC is a *Right Angle Triangle*, Right-angled at B.---In all *Right Angle Triangles*, the longest Leg is called the *Hypothenufe*;---the Leg on which it stands, the *Base*;---and the other Leg, the *Perpendicular*.



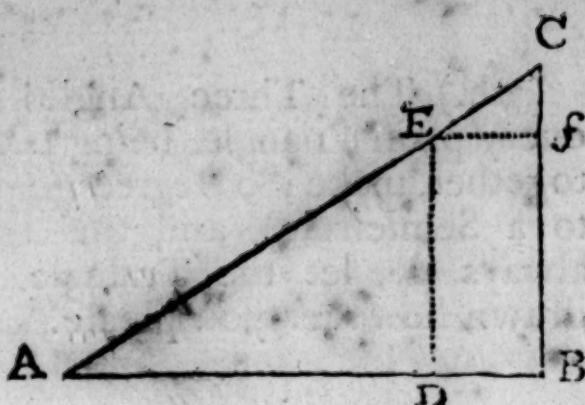
(9th.) If neither of the Angles is a *Right One*, then, it is called an *Oblique Angled Triangle*; as the Triangle CDE is an *Oblique Triangle*.



B

(10th.) If

(10th.) If the Angles of one Triangle are equal to the Angles of another Triangle, the Sides of the former are proportioned to the Sides of the latter.



Thus, If in the Triangle ABC, you draw the Line ED parallel to CB, the smaller Triangle ADE will be *similar to*, i. e. will have the same Angles with the larger Triangle ABC.---It will therefore always hold, as

$$\begin{aligned} &AB : AD :: BC : DE. \\ \text{Or, as } &AB : BC :: AD : DE. \\ \text{Or, as } &AD : DE :: AB : AC. \\ \text{Or, as } &AD : DE :: Ef : fC, \text{ \&c.} \end{aligned}$$

(11th.) In all Triangles the *greatest Side* is opposite to the *greatest Angle*; and on the contrary, the *greatest Angle* is opposite the *greatest Side*.---If Two Sides are *equal*, the opposite Angles are *equal*.---If all the Sides are equal, then all the Angles are equal to each other.

(12th.) In all Triangles, every Side is in proportion to its opposite Angle, and every Angle to its opposite Side: And further, as the Angle opposite to one Side, is to the Angle opposite the other Side, so are the Sides themselves one to another; and the contrary, the Sides to the Angles.

Every Triangle, as I observed before, consists of Six Parts---Three Sides and Three Angles. If any Three of the Six Parts (excepting the Three Angles) are given, any one, or every one of the rest may be found, without the painful Deductions and voluminous Tables of *Logarithms, Sines, Tangents, and Secants*, by the following Rules and Axioms*.

* This Method will be found as exact, as that by the *Logarithms*, if you carry on the Operation to Three or Four Decimal Places; but for common Purposes, One or Two Decimal Places will be near enough. You must also remember to reduce the *Minutes* (and *Seconds*) of the Angles to *Decimals* of a Degree, which is easily done, by allowing One Tenth for every Six Minutes.---Or you may turn the Minutes into Decimals thus: As 60, the Minutes in one Degree, : are to the Minutes given, : : so are 10, 100, 1000, &c. to the Decimal required.

The foregoing Properties of a Triangle are so manifest, that they stand in need of no further Illustration or Demonstration.

Of Right Angled TRIANGLES.

THERE are generally reckoned by Writers on this Subject Seven Cases; but by this Method they are all reduced to Four; the Solutions of which depend on the following *Axioms*.

AXIOM I. Divide 4 Times the Square of the Complement of the Angle, whose opposite Side is either given or sought, by 300 added to 3 Times the said Complement; this Quotient added to the said Angle, will give you an *Artificial Number*, called sometimes the *Natural Radius* *, which will ever bear the same Proportion to the Hypotenuse, as that Angle bears to its Side.—In Angles under 45 Degrees, the *Artificial Number* may be found thus: Divide 3 Times the Square of the Angle itself, whose opposite Side is given or sought, by 1000; the Quotient added to 57.3 †, a fixed Number, that Sum will be the *Artificial Number* required.—This is to be used, when the Angles and a Side are given, to find another Side.

This Axiom is derived from collecting together a few leading Terms of a swift converging Series for determining the Length of the Sine or Co-sine, from the Length of a given Arch. Thus, If x represent the versed Sine, and z the Length of the correspondent Arch, the Radius being r , we shall have $x = \frac{x^2}{2r} - \frac{x^4}{24r^3}$ (see Simpson's Fluxions, p. 500) and, consequently, the Cosine of the Arch $z = r + \frac{x^4}{24r^3} - \frac{x^2}{2r}$. Now if we suppose $r = 1$, the said Co-sine will be $1 + \frac{x^4}{24} - \frac{x^2}{2}$, which gives this rule: "Square the Complement of the given Arch. " and divide it by 2, subtract one Sixth of the Quantity squared from " the Quotient, and the Remainder from the Radius = 1. the Resi- " due will be the Sine of the proposed Arch." This Rule will give the Sine true to three Places in Decimals; but the Length of the Arch measuring the Angle must be taken in Terms of the Radius: In Order to remove this Difficulty let the Radius be increased to 57.3 nearly equal to the Radius of a Circle, whose Circumference is 360 equal Parts. Substitute this Value for r in the above Expression, $r - x = r + \frac{x^4}{24r^3} - \frac{x^2}{2r}$, and we shall have $57.3 + \frac{x^4}{24 \times 57.3^3} - \frac{x^2}{2 \times 57.3}$ for the Cosine of the Arch z , which may now be expressed by the Degrees and Decimal Parts, measuring the given Angle.

* The *Natural Radius* is only turning the *Right Angle*, $\hat{=}$ 90 Degrees, into an *artificial Number*, which shall always bear the same Proportion to the *Hypotenuse*, as the given Angle does to its opposite Leg.

† 57.3 is the *Radius* of a Circle whose Circumference is 360, or more exact, 57 25979.

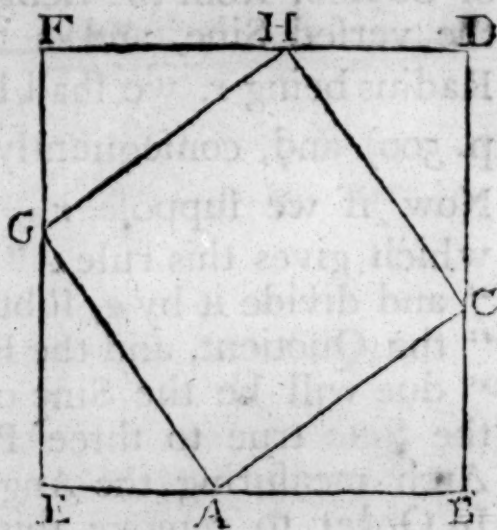
From this Investigation the first Part of the above Axiom is easily deduced, as will appear by expressing that Rule algebraically, in Terms of the Radius 57.3, z being the Co-sine of the given Angle; for it will stand thus: $\frac{4z^2}{300+3z} + 90 - z$, the Radius; consequently, it will be, as $90 - z : \frac{4z^2}{300+3z} + 90 - z ::$ the Sine of the Angle expressed by $90 - z : 57.3$, the Radius of a Circle whose Circumference is 360 equal Parts. *Q. E. D.*

The second Part of this Axiom may be easily investigated in the following Manner: Let z = the Length of any Arch, expressed in Terms of r , the Radius, and y = the Sine of that Arch; then will $y = z - \frac{z^3}{2.3r^2} + \&c.$ And if we put $r = 1$, then will $y = z - \frac{z^3}{2.3}$; or, which is the same Thing $y = 1 - \frac{z^2}{6} \times z$, be the Sine of that Arch expressed in Terms of the Radius. (See Simpson's Fluxions, p. 501) But in Order to express it in Terms of the Angle itself, we must put $r = 57.3$, the Radius of a Circle whose Circumference is 360 equal Parts. Then will the above Series, when the Angle is less than 45 Degrees, give an Expression nearly to $\frac{3z^2}{1000} + r = y$, the Rule laid down in the second Part of this Axiom. *Q. E. D.*

AXIOM II. The Square of both the Legs, *i. e.* the Square of the Base and Perpendicular added together, is equal to the Square of the Hypotenuse; whose Root is the Hypotenuse itself.—*This is made use of, when the Base and Perpendicular are given, to find the Hypotenuse.*

This is the 47th Proposition of Euclid's 1st Book, and may be easily demonstrated in the following Manner.

Let ABC be a Right Angled Triangle, and let the Base AB = a , the Perpendicular BC = b , and the Hypotenuse AC = x ; then will $xx = aa + bb$.



DEMONSTRATION.

Continue the Perpendicular BC till $CD = AB$, and on BD, as a Base, draw the Square BDFE. Upon AC, as a Base form another Square, as ACHG. Then will the Area of the great Square, BDFE = the Area of the four Triangles ABC, CDH, HFG, GAE + the Area of the Square ACHG. But the Area of each Triangle is $\frac{ab}{2}$; consequently, the Area of the four Angles is $2ab$. Therefore, the Area of the great Square is $xx + 2ab$. But the Area of the great Square is also = the Rectangle of $a + b$, or $aa + 2ba + bb$; consequently, $xx + 2ba = aa + 2ba + bb$. Expunge $2ba$ from both Sides of the Equation, and it will be $xx = aa + bb$. *Q. E. D.*

AXIOM

AXIOM III. The *Sum* of the *Hypothenuſe* and *One* of the *Legs* multiplied by their *Difference*, the *Square Root* of that Product will be the *other Leg* required.---This comes into uſe, when the *Hypothenuſe* and *One Leg* is given, to find the *other Leg*.

This is only a Consequence of the last Demonstration; for it is there proved, that

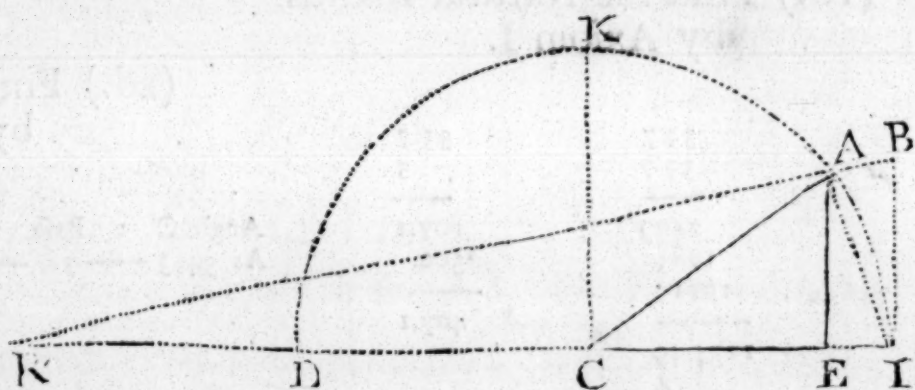
$aa + bb = xx$. Consequently $xx - bb = aa$.

But $x + b \times x - b = xx - bb.$ *Q. E. D.*

AXIOM IV. *Half the Longer of the Two Legs, added to the Hypothenuse, is always in Proportion to 86^{*}, as the Shorter Leg is to its opposite Angle.---This is useful, when the Sides are given, to find the Angles.*

CONSTRUCTION.

Let CAE be the Triangle proposed. With the Radius CA = the Hypothenufe, describe the Semi-circle IKDC, and, producing the Dia-



meter towards R, make $DR = DC$, the Radius. Draw the Tangent IB, and the Chord IA. Then will the Chord IA be equal to, or sufficiently near equal to IB, the Tangent of the Angle ACE; and RI is Triple of the Radius CI = the Hypothenufe CA.

DEMONSTRATION.

Supposing $IB = IA$; the two Triangles RIB , and REA are similar, because AE is parallel to BI . Therefore it will be (by the 6th Euclid's 4th.)

As $RE : EA :: RI : IB = IA$. And,

$$\text{As RE : EA :: RI : IA} = \text{IB.}$$

The Chord IA must be substituted for an Arch of a Circle whose Circumference = 360 equal Parts, whose Half is DKI. But the Radius of a Circle whose Circumference is 360, is 57.2 957 = CI, which tripl'd is 171.8871, or 172 nearly, = RI, by Construction; and RD = DC = CA, the Hypothenuse: Therefore it will be, as RE = RD + DC + CE, or twice the Hypothenuse, + CE the Base, is to RI, thrice the Radius = 172; so is EA, the shortest Leg to IB = IA = the Angle ACE, required. Or, more briefly, by taking Half the two first Terms, it will be,

As $CA + \frac{CE}{2} : \text{to } \frac{RI}{2} = 86 :: \text{fo is AE the shortest Leg: to its opposite Angle ACE required. } Q. E. D.$

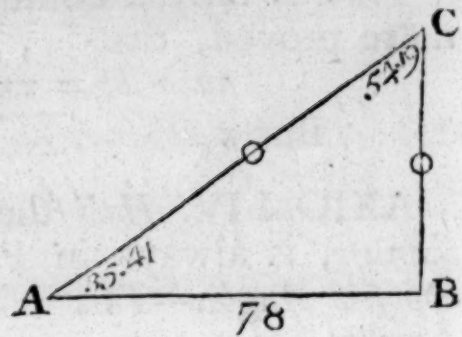
NOTE. These four *Axioms* will answer all the Cases of *Right* and *Oblique Angled Triangles*, except the last Case in *Obliques*, which will require some further Assistance, and will be shewn when we come to treat of that Case.

* 86 = Radius and Half of a Circle whose Circumference is 360, or, 85.94368.

CASE I.

The Acute Angles, and one Leg given; to find the Hypothenufe and the other Leg.

Given $\left\{ \begin{array}{l} \text{Angle A} = 35.41 \\ \text{Angle C} = 54.19 \\ \text{Base AB} = 78 \end{array} \right\}$ find $\left\{ \begin{array}{l} \text{Hypothenufe AC} \\ \text{and} \\ \text{Perpendic. BC.} \end{array} \right.$



(1st.) Find the Natural Radius by Axiom I.

$$\begin{array}{r}
 35.7 \\
 35.7 \\
 \hline
 2499 \\
 1785 \\
 1071 \\
 \hline
 1274.49 \\
 4 \\
 \hline
 407.1 \quad 5097.96 \quad (12.5 \\
 4071 \quad (54.3 \\
 \hline
 10269 \quad 66.8 \text{ Natural Radius} \\
 8142 \\
 \hline
 21276 \\
 20355 \\
 \hline
 921
 \end{array}$$

(2d.) Find the Hypothenufe by Axiom I.

$$\begin{array}{r}
 \text{Angle C} : \text{Base} :: \text{Nat. Rad.} \\
 \text{As } 54.3 \text{ --- } 78 \text{ --- } 66.8 \\
 \hline
 78 \\
 \hline
 5344 \\
 4676 \\
 \hline
 54.3 \quad 5210.4 \quad (95.9 + \text{Hypothenufe} \\
 4887 \\
 \hline
 3234 \\
 2715 \\
 \hline
 5190 \\
 4887 \\
 \hline
 303
 \end{array}$$

(3d.) Find the Perpendicular by Axiom III.

$$\begin{array}{r}
 96 \text{ To Hypothenufe} \\
 78 \text{ Add the Base} \\
 \hline
 174 \text{ Sum multiply} \\
 18 \text{ by Difference} \\
 \hline
 1392 \\
 174 \\
 \hline
 \text{Extract the Root } 3132 (55.9 + \text{Perpendicular} \\
 25 \dots \\
 \hline
 105 \quad 632 \\
 525 \\
 \hline
 1109 \quad 10700 \\
 9981 \\
 \hline
 719
 \end{array}$$

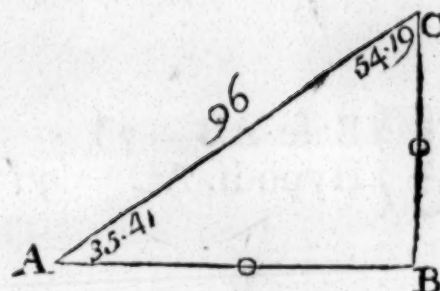
Answer, $\left\{ \begin{array}{l} \text{Hypothenufe, } 95.9 +, \text{ or } 96. \\ \text{Perpendicular, } 55.9 +, \text{ or } 56. \end{array} \right.$

CASE

CASE II.

The Hypothenufe and Angles given, to find the Two Legs.

Given { Hypoth. AC = 96 } to find { Perpendic. CB
Angle A = 35.41 and
Angle C = 54.19 } Base AB.



(1st.) Find the Natural Radius
by Axiom I.

```

35.7
35.7
---
2499
1785
1071
---
1274.49
3
---
1.000 3.823.47
57.3 Add
---
61.1 Natural Radius
    
```

(2d.) Find the Perpendicular
by Axiom I.

Nat. Rad.	:	Hypoth.	::	Angle A
As 61.1	—	96	—	35.7
				96
				2142
				3213
				61.1)3427.2(56 Perpen
				3055
				3722
				3666
				56

(3d.) Find the Base by Axiom III.

```

96 To Hypothenufe
56 Add Perpendicular
---
152 Sum multiplied by
40 The Difference
---
Extract the Root 6080)77.9 + Base
49
---
147)1180
1029
---
1549)15100
13941
---
1159
    
```

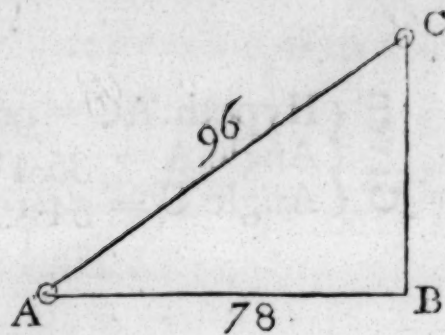
Answer, { Perpendicular, 56.
Base, 77.9 +, or 78.

CASE

CASE III.

The Hypotenuse and One Leg given, to find the Angles and the other Leg.

Given $\left\{ \begin{array}{l} \text{Base } AB = 78 \\ \text{Hypoth. } AC = 96 \end{array} \right\}$ find $\left\{ \begin{array}{l} \text{Perpendic. } CB \\ \text{and} \\ \text{Angles } A \text{ \& } C. \end{array} \right.$



(1st.) Find the Perpendicular by Axiom III.

96 To Hypotenuse
78 Add the Base

174 Sum multiply
18 By Difference

1392

174

Extract the Root 3132 (55.9 + Perpendicular)

25

105)632

525

1109)10700

9981

719

(2d.) Find the Angle by Axiom IV.

To Hypotenuse 96
Add half longer Leg 39

Sum 135

: Fixed Number

86

: : Perpendicular

56

86

336

448

135)4816. (35.67 + Angle A.

405

766

675

910

810

1000

945

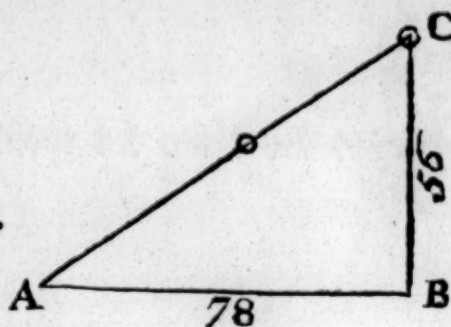
55

Answer, $\left\{ \begin{array}{l} \text{The Angle, } 35^\circ 41' \text{ nearly.} \\ \text{The Perpendicular, } 55.9 +, \text{ or } 56. \end{array} \right.$

CASE IV.

The Two Legs given; to find the Hypothenufe and the Angles.

Given { Base AB = 78 } find { Hypoth. AC.
Perpend. CB = 56 } to { and
Angles A & C.



(1st.) Find the Hypothenufe by Axiom II.

Perpendicular 56	78 Base
squar'd 56	78 squar'd
<u>336</u>	<u>624</u>
280	546
<u>3136</u>	6084 Square of Base
	3136 Square of Perpendic.

Extract the Root 922096 The Hypothenufe

81
186(1120
1116
4

To Hypothenufe 96
Add half longer Leg 39
Sum 135

: Fixed Number :: Perpendicular

56
86
336
448
135)4816.(35.674 + Angle A
405
766
675
910
810
1000
945
550
540
10

Answer, { The Hypothenufe, 96.
The Angle, 35°.674, or 35° 41' nearly.

D

NOTE.

NOTE. Thus all the Cases of *Right Angled Triangles*, are easily and readily answered: and by the same Rules, and with the like Ease may the *Oblique Angled Triangles* be answered, as will evidently appear in the following Cases.

Of Oblique TRIANGLES.

IN the Solution of an *Oblique Triangle*, it is necessary, by this Method, to divide it into Two *Right Angled Triangles*, by means of a *Perpendicular*, which must always fall from the End of a given Side, and opposite to a given Angle.

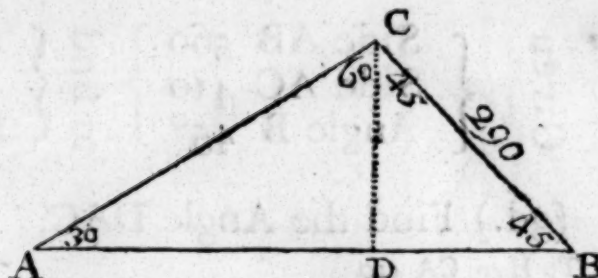
By this means the *Perpendicular* will sometimes fall *within*, and sometimes *without* the Triangle: When it falls within, it falls upon some Part of the Base, or longest Side; but when it falls without, it falls upon one of the shorter Sides continued. In either Case, there are Two *Right Angled Triangles* made, and the Angles, or Sides sought, are found as if they were Parts of a *Right Angled Triangle*, by the foregoing Axioms; but it requires Two or Three Operations.

CASE

CASE I.

Two Angles, and a Side opposite to one of them, given; to find the other two Sides.

Given { Angle A 30 { find } Side AC
Angle B 45 { } and
Side BC 290 { to } Side AB.



(1st.) Find the Perpendicular CD.

N. Rad. Hypoth. \angle at B
As 63.6 ——— 290 ——— 45

1450
1160

63.6) 13050 (205 Perpendicular CD.
1272

3300
3180
120

(2d.) Find the Side AC.

\angle at A Perp. CD N. Rad.
As 30 ——— 205 ——— 60

30) 12300 (410 The Side AC Hypotenuse.
120

30
30
0

(4th.) Find the Side DB.

To Hypoth. CB 290
Add Perpend. CD 205

Sum 495
Multiply by Difference 85

2475
3960

Extract the Root 42075 (205 DB.

4
405) 2075
2025
50

45 45
45 3
225 135
180 300
2025 435
4

435) 8100 (18.6
435 45

3750 63.6 Natural Radius
3480

2700 30
2610 30
90 900
3

2700
57.3

Natural Radius 60.0

(3d.) Find the Side AD.

To Hypoth. AC 410
Add Perpend. CD 205

Sum 615
Multiply by Difference 205

3075
12300

Extract the Root 126075 (355 AD

9
65) 360
325

705) 3575
3525

50

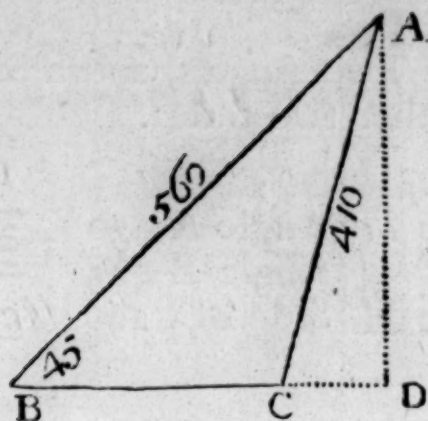
Answer, { The Side AC 410.
The Side AB 560.
The Angle C 105.

CASE

CASE II.

Two Sides, and an Angle opposite to One of them, being given; to find the rest.

Given $\left\{ \begin{array}{l} \text{Side AB } 560 \\ \text{Side AC } 410 \\ \text{Angle B } 45^\circ \end{array} \right\}$ find $\left\{ \begin{array}{l} \text{Angle C} \\ \text{and} \\ \text{Side BC.} \end{array} \right.$



(3d.) Find the Angle DAC.

To Hyp. CA 410
Add $\frac{1}{2}$ Perp. AD 198

As 608 ———— Fix'd Num. Side CD

86 ———— 106
86
636
848
608)9116(14.9+, or $15^\circ \angle A$
608
3036
2432
6040
5472
568

(1st.) Find the Side AD.

Nat. Rad. Hyp. BA $\angle B$
As 63.6 ———— 560 ———— 45
45
2800
2240
63.6, 25200.0)396 AD
1908
6120
5724
3960
3816
144

(4th.) Find Side BD.

To Hypoth. BA 560
Add Perpend. DA 396

Sum 956
Multiply by Differ. 164

3824
5736
956

Extract the Root $156784(395.9 + \text{ or } 396 \text{ BD}$
9..

69 667
621

785(4684
3925

7909)75900
71181

4719

Then from BD = 396
Take ——— CD = 106

Remains — BC = 290 Required. and The Angle BAC = 30° Requ.

In this, and several of the following Cases and Problems, the Operation for finding the Natural Radius is omitted, for Want of Room.

(2d.) Find CD.

To Hypoth. CA 410
Add Perpend. AD 396

Sum 806
Multiply by Difference 14

3224
806

Extract the Root $11284(106 \text{ CD}$
1

206(1284
1236
48

Hence we find by Inspection.

The Angle ACD = 75°

The Angle ACB = 105°

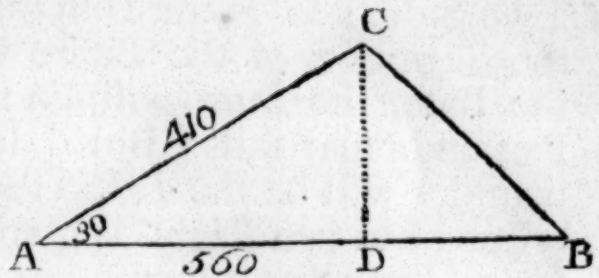
The Angle BAC = 30° Requ.

CASE

CASE III.

Two Sides, with the Angle comprehended by them, given; to find the rest.

Given } Side AC 410 } find { Side BC
Side AB 560 } and
Angle A 30° } Angles B and C.



(1st.) Find the Perpendicular CD.

N. Rad. Hypoth. AC \angle at A
As 60 — 410 — 30
30
60)12300(205 Perpendicular CD
120
300
300
0

(2d.) Find the Part AD.

To Hypoth. AC 410
Add Perpend. CD 205
Sum 615
Multiply by Differ. 205
3075
12300
Extract the Root 126075(355 Base AD
9....
65)360
325
705)3575
3525
50

From AB = 560
Take AD = 355
Remains BD = 205

(3d.) Find the Side BC.

BD squar'd 205 205 CD squar'd
205 205
1025 1025
4100 4100
To Square of DB 42025
Add Square of CD 42025
Extract the Root 84050(289.9 +, or 290 BC
4
48)440
384
569)5650
5121
5789)52900
52101
799

(4th.) Find the Angle B.

To 290 the Side BC
Add 102.5 half DB
Fix'd Num. Side DC
As 392.5 — 86 — 205
86
1230
1640
392.5)17630.0(44.9 +, or 45°
15700
19300
15700
36000
35325
675

The Angle A = 30, added to Angle B = 45, and then subtracted from 180, leaves 105 for Angle C.

Answer, { The Side BC 290
The Angle B 45°
The Angle C 105°.

E

CASE

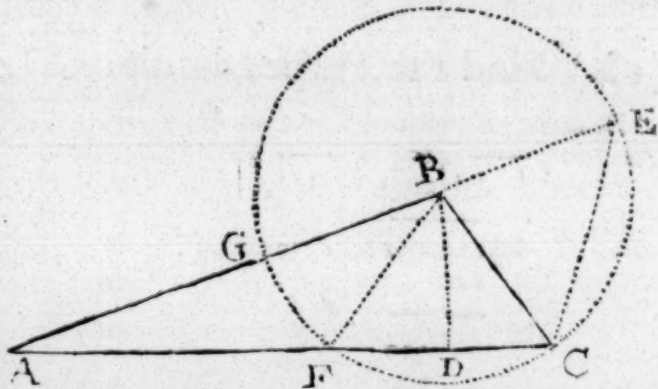
CASE IV.

The Three Sides given; to find the three Angles.

In all Triangles, as the Base or greater Side is to the Sum of the other Two Sides; so is the Difference of the Sides to the Difference of the Segments of the Base; which Half Difference, added to Half the Base, the Sum will be the *Greater Segment*, upon which the Perpendicular falls: But if subtracted from Half the Base, the Remainder will be the *Less Segment*: So will the *Oblique Triangle* be reduced to two Right Angled ones, and may be answered after the same Manner as before.

CONSTRUCTION.

Let ABC be the Triangle proposed, whose three Sides AB, AC, and BC are given. On B, as a Centre, with the Radius BC, describe the Circle GFCEB; continue the Line AB till it cuts the Circle in E; let fall the Perpendicular BD; draw the Lines EC and BF. Then will AE be the Sum of the two Sides AB and BC, and AG, their Difference. Also $AC = AD + DC$, the Sum of the Segments of the Base, and $AF = AD - FD$, their Difference.



DEMONSTRATION. Because the Triangles ABF, and AEC, are similar, it will be, as $AC : AE :: AG : AF$. Q. E. D.

Given $\left\{ \begin{array}{l} \text{Side AB } 560 \\ \text{Side AC } 410 \\ \text{Side BC } 290 \end{array} \right\} \left\{ \begin{array}{l} \text{find} \\ \text{Angle A} \\ \text{Angle B} \\ \text{Angle C.} \end{array} \right.$

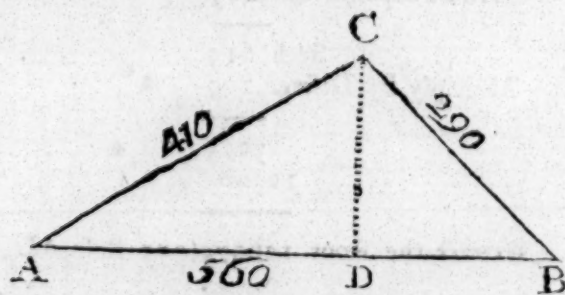
(1st.) Find the Segments of the Base.

As 560	700	120
	120	
	14000	
	700	
	560	84000 (150 Difference)
To $\frac{1}{2}$ Base 280	560	
Add $\frac{1}{2}$ Diff. 75	2800	
G. Seg. AD 355	2800	
	0	
From $\frac{1}{2}$ Base 280		
Subtract $\frac{1}{2}$ Diff. 75		
Lesser Seg. DB 205		

(2d.) Find the Perpendicular CD, by Case III. = 205.

Then the Angle $A = 30^\circ$; added to $B = 45^\circ$, and subtracted from 180° , leaves 105° for the Angle C, which were the Angles required.

THE



(3d.) Find the Angle at A.

To 410 Side AC	
Add 177 half AD	
As 587	86
Fix'd Num.	Side DC
205	30 \angle A

(4th.) Find the Angle at B.

To 290 Side BC	
Add 102 half DB	
As 392	86
Fix'd Num.	Side DC
205	49.9+, or 45 \angle B

THE USE OF TRIGONOMETRY

Exhibited in the Solutions of a Number of interesting Problems; many of which every Day occur; are of the greatest Utility in the Army, Navy, &c. and cannot be answer'd without it.

PROBLEM I.

To take the Height of any *accessible Object* at one Station.

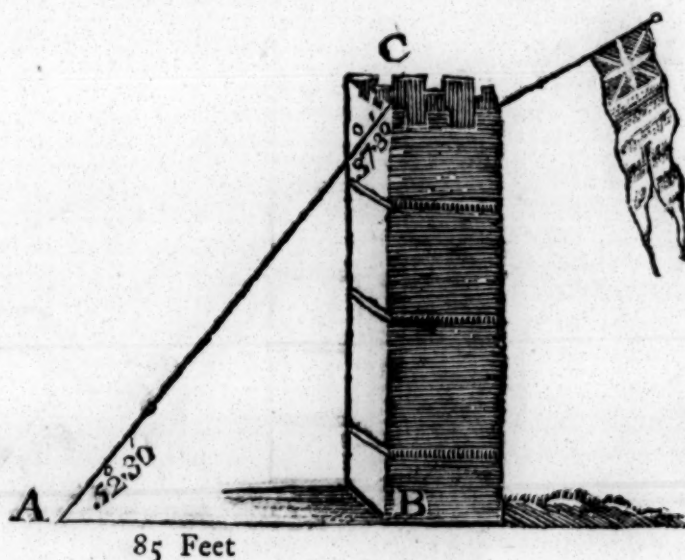
First, with a *Quadrant*, by looking through the Sights to the Top of the Tower, find the Quantity of the Angle A, which suppose $52^{\circ} 30'$; then measure the Distance AB, which suppose to be 85 Feet; from hence you may proceed to find the *Height*, by Case I. of Right Angled Triangles.

$$\begin{array}{r} 37.5 \\ 37.5 \\ \hline 1875 \\ 2625 \\ 1125 \\ \hline 1406.25 \\ 3 \end{array}$$

$$\begin{array}{r} 1000)4.218.75 \\ 57.3 \end{array}$$

61.51875 Natural Radius

But 61.52 is exact enough in Practice.



(2d.) Find the Perpendicular BC.

(1st.) Find the Hypothenuse AC.

Angle C : Base :: Nat. Rad.
As 37.5 — 85 — 61.52

$$\begin{array}{r} 85 \\ 30760 \\ 49216 \end{array}$$

37.5)5229.20.(139.44+Hyp.

$$\begin{array}{r} 375 \\ 1479 \\ 1125 \\ \hline 3542 \\ 3375 \\ \hline 1670 \\ 1500 \\ \hline 1700 \\ 1500 \\ \hline 200 \end{array}$$

To Hypothenuse 139.44
Add Base 85.

Sum 224.44
Multiply by Difference 54.44

$$\begin{array}{r} 89776 \\ 89776 \\ 89776 \\ 112220 \end{array}$$

Extract the Root 12218.5136(110.537 Perpend.

$$\begin{array}{r} 1 \dots \dots \dots \\ 21)22 \\ 21 \\ \hline 2205)11851 \\ 11025 \\ \hline 22103)82636 \\ 66309 \\ \hline 221067)1632700 \\ 1547469 \\ \hline 85231 \end{array}$$

Answer, 110.537 +, or 110 Feet, and above $\frac{1}{2}$: The Height required.

NOTE; That in this, and all such Cases, you must add the *Height* of your *Eye*, or *Instrument*, to the *Altitude* before found.

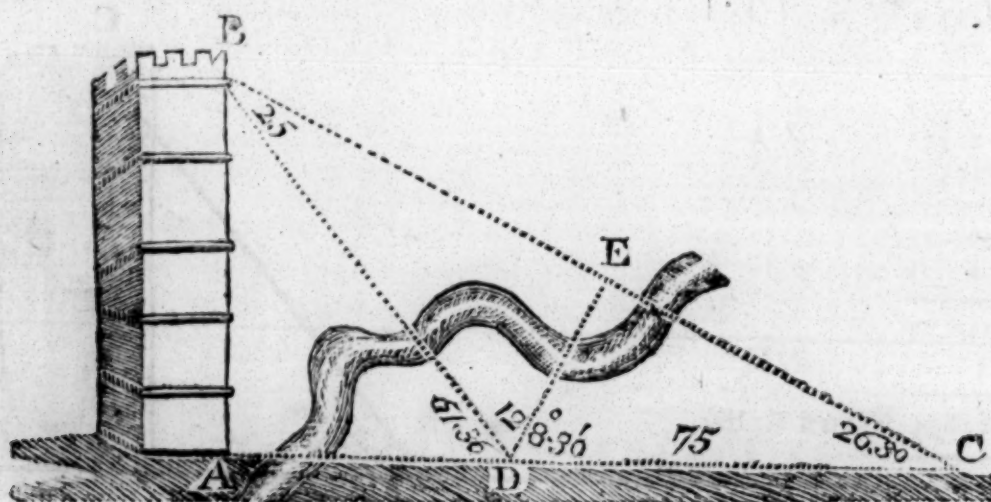
PROBLEM

PROBLEM II.

To measure an *Inaccessible Altitude*.

Let AB, in the following Figure, be a *Church, Tower, or Fort*, whose Height is required; but by Reason of a River, or some other *Obstacle*, it is *inaccessible*; that is, you cannot come to the Foot of it, by Reason of the *Water, &c.*

First, with a *Quadrant*, take the Angle of *Altitude* at C, which suppose $26^{\circ} 30'$. Then measure in a Right Line towards the *Tower* to D, any Distance, suppose 75 Feet, and at D observe again the Angle of *Altitude*, which let be $51^{\circ} 30'$.



Then; the two Visual Lines CB and DB, with the Distance DC, make the Oblique Triangle CBD, in which are given—All the Angles and Side CD. The Angles BCD being $26^{\circ} 30'$, and the Complement of ADB $51^{\circ} 30'$ to 180, is the Obtuse Angle BDC $128^{\circ} 30'$. Consequently, the third Angle CBD, at the Top, is $= 25^{\circ}$.

(1st.) Find the Perpendicular DE in Triangle DBC.

$$\begin{array}{l} \text{Nat. Rad. : Op. Side DC :: Ang. C : Perp. DE} \\ \text{As } 59.4 \text{ — } 75 \text{ — } 26.5 \text{ — } 33.46 \end{array}$$

(3d.) Find the Height AB in the Right Angled Triangle ABD.

$$\begin{array}{l} \text{Nat. Rad. : Op. Side BD :: Ang. D : Height} \\ \text{As } 65.7 \text{ — } 79.19 \text{ — } 51.5 \text{ — } 62 \text{ AB} \end{array}$$

(2d.) Find the Visual Line BD in Triangle BDE.

$$\begin{array}{l} \text{Ang. B : Op. Side DE :: N. Rad. : Side BD} \\ \text{As } 25 \text{ — } 33.46 \text{ — } 59.17 \text{ — } 79.19 \end{array}$$

(4th.) Find the Distance AD in the Triangle ABD.

$$\begin{array}{l} \text{N. Rad. : Op. Side BD :: Ang. ABD : Dist. AD} \\ \text{As } 61.7 \text{ — } 79.19 \text{ — } 38.5 \text{ — } 49.41 \end{array}$$

Answer, } 62 Feet the Height.
 } 49.41, or $49\frac{1}{2}$ Feet the Distance from the second Station.

NOTE: The Line BD is the Length of a *Scaling Ladder*, which would reach from the *Station* at D over the *Foss* or *Ditch*, to the *Top* of the *Tower* at B.

PROBLEM

P R O B L E M III.

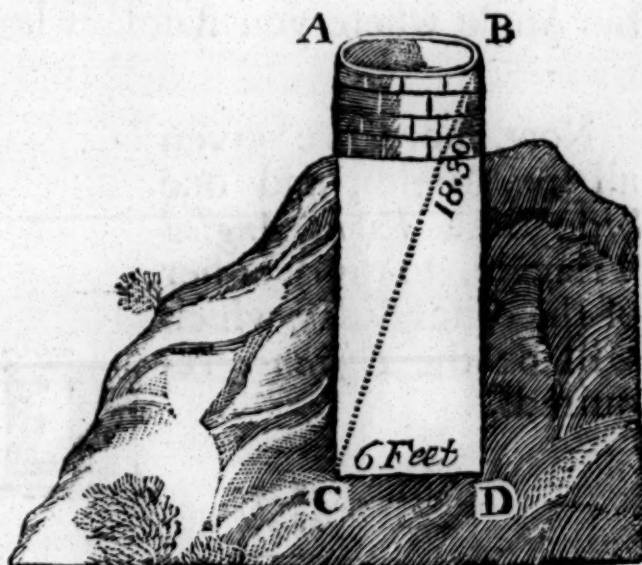
To measure the *Depth* of a *Well*, or the *Height* of an *Object* from the *Top* of it.

First, look through the *Sights* of the *Quadrant* to the *Bottom* of the opposite Side the *Well* at C, so you will have the *Angle* CBD; next, take the *Breadth* AB at the *Top*, which is equal to CD at the *Bottom*: Then, by Case I. of Right Angle Triangles, you may easily find the *Depth* BD required.

Suppose the *Angle* at B, by *Observation*, to be $18^{\circ} 30'$, and the *Breadth* at the *Top* 6 Feet: What's the *Depth*?

$$\begin{array}{r}
 18.5 \\
 18.5 \\
 \hline
 925 \\
 1480 \\
 185 \\
 \hline
 342.25 \\
 3 \\
 \hline
 1.026.75 \\
 57.3 \\
 \hline
 \end{array}$$

Natural Radius 58.32675 but 58.3 is enough.



(1st.) Find the Hypothenufe BC.

$\angle B$	Op. Side	N. Rad.
As 18.5	— 6	— 58.3
		6

$$\begin{array}{r}
 18.5)349.8.(18.9 + \text{the Line BC} \\
 185 \\
 \hline
 1648 \\
 1480 \\
 \hline
 1680 \\
 1665 \\
 \hline
 15
 \end{array}$$

Answer, { 17.89 +, or 18 Feet, the
Depth required.

(2d.) Find the Depth BD.

To Hypoth. BC	18.9
Add Side CD	6.

Sum	24.9
Multiply by Differ.	12.9

$$\begin{array}{r}
 2241 \\
 498 \\
 249 \\
 \hline
 \end{array}$$

Extract the Root 321.21 (17.89 the Depth AB)

$$\begin{array}{r}
 27)221 \\
 189 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 348)3221 \\
 2784 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3569)43700 \\
 32121 \\
 \hline
 11579
 \end{array}$$

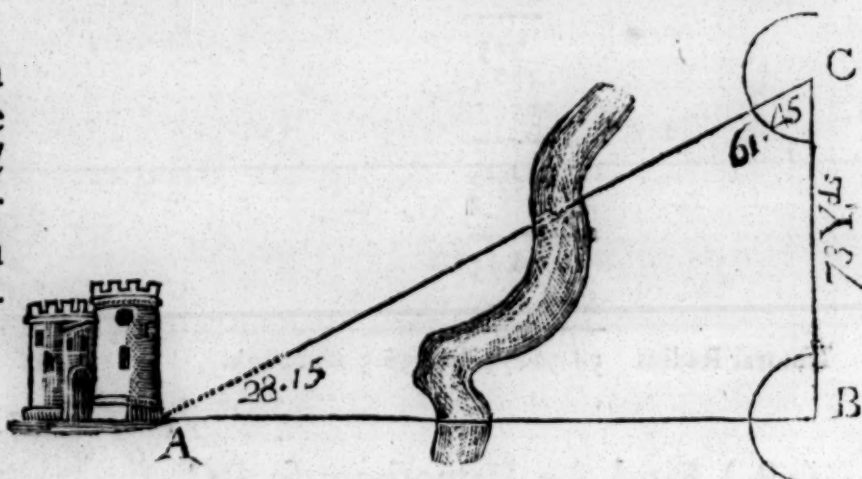
PROBLEM IV.

To measure the *Distance* of any *Object*.

Suppose yourself standing at B, and a great Way off, as at A, you see a *Fort* or *Castle*, &c. or any other *Object*, whose *Distance* you would find from the Place where you now stand.

First, a *Theodolite*, or *Semicircle*, being placed at B, lay the *Index*, with its *Sights*, on the *Diameter*, where the Degrees begin, and through them view the *Castle*, &c. at A. The Instrument remaining fix'd in this Position, move the *Index* to 90 Degrees, (that being a Right Angle) and view some Mark at a Distance, (the farther off the better) as at C. → Next measuring the Distance from B to C, which suppose 73 Yards, remove your Instrument, and set it up at C. Then, with the *Index* laid upon the Beginning of the Degrees, as before, turn the Instrument about, till you can see your *first Station* at B, where fasten it; then turn the *Index* till you can see the *Object* A, and observe what Degrees are cut, as suppose $61^{\circ} 45'$, which is the Quantity of the Angle where you stand; whose Complement to 90° is the Angle A.

Now, here are given all the Angles, and one Side of a *Right Angled Triangle*, to find either of the other Sides, which will be the *Distance* required.



$$\begin{array}{r}
 28.25 \\
 28.25 \\
 \hline
 14125 \\
 5650 \\
 22600 \\
 5650 \\
 \hline
 798.0625 \\
 3 \\
 \hline
 23941875 \\
 57.3 \\
 \hline
 59.69 \text{ or } 59.7 \text{ Natural Radius}
 \end{array}$$

(2d.) Find the Distance from B.

$$\begin{array}{r}
 \text{To Hypoth. AC } 154.2 \\
 \text{Add Side BC } 73 \\
 \hline
 \text{Sum } 227.2 \\
 \text{Multiply by Differ. } 81.2 \\
 \hline
 4544 \\
 2272 \\
 \hline
 18176
 \end{array}$$

(1st.) Find the Distance from C. Extract the Root $\sqrt{18448.64} (135.8 +, \text{ Dist. from B})$

$$\begin{array}{l}
 \text{Ang. A : Op. Side BC :: N. Rad. : Dist. CA} \\
 \text{As } 28.25 - 73 - 59.7 - 154.2
 \end{array}$$

yards
 Ans. } $135.8 +$, or 136 Dist. from B.
 } 154.2 , or $154\frac{1}{2}$ Dist. from C.

$$\begin{array}{r}
 1 \\
 23 \overline{) 84} \\
 69 \\
 \hline
 265 \overline{) 1548} \\
 1325 \\
 \hline
 2708 \overline{) 22364} \\
 21664 \\
 \hline
 700
 \end{array}$$

PROBLEM

P R O B L E M V.

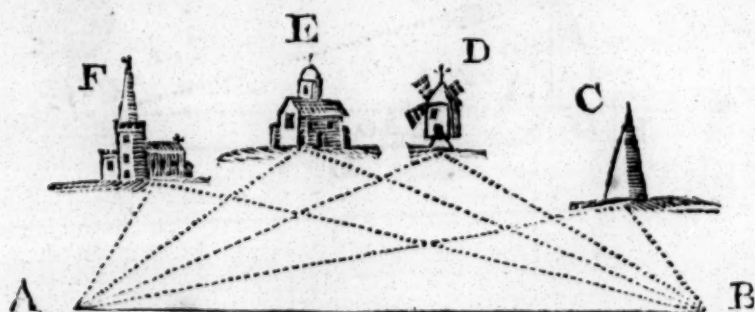
To take the *Distances* of several *inaccessible Objects*, as *Forts, Churches, in a Town, or Squadron of Ships at Sea*, and to delineate them upon Paper.

First, make choice of two places, from either of which you may conveniently see all the *Objects*; which two Places let be A and B in the following Figure.—This being done, set up your Instrument at A, laying the *Index* on the *Diameter*, and turn the whole Instrument about, till, through the Sights, you see your second Station B. Then, fixing the Instrument, direct your Sights to the several *Objects*, C, D, E, and F; noting down the Degrees cut at each Observation, which suppose to be as in the Table.

Then, remove the Instrument to B, laying the *Index* on the *Diameter*, and turn it about, till, through the Sights, you see your former Station at A; then direct your Sights to every one of the Objects at C, D, &c. setting down the Degrees at each Observation, as in the Table. Also measure the *Stationary Distance*, and set that down.

	C	D	E	F
1st Station	11	23	36	59
2d Station	51	31	22	13
Stationary Distance 150 Yards.				

First, upon a Piece of Paper draw the Line AB; and from a *Scale of equal Parts*, take off, with your Dividers, the *Stationary Distance* = 150, and set it from A to B, so will A represent your *first Station*, and B the *second*. Then lay the *Center* of the *Protractor* upon the



Point A, with its *Diameter* upon the Line AB; keeping it fast, make Marks by the Edge at 11, 23, 36, 59, and draw Lines from the Point A through each of those Marks. Then upon B place the *Center* of your *Protractor*, its *Diameter* lying upon the Line AB; make Marks by the Side at 13, 22, 31, 51. Then draw Lines from the Point B through each of these Marks, and where the Lines cut the former *correspondent Lines* there will be found the Places representing these *Objects*. Then any of these Lines being taken in a Pair of Dividers, and applied to the Scale you laid your *Stationary Distance* down by, will give you their *Distances*, either from your *Stations* or from one another.

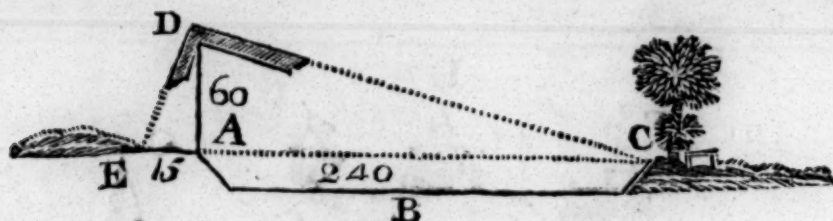
The *Distance* of any of these *Objects* from either *Station*, &c. may be found by *Calculation*; one *Side* and the *Angles* being given: But I shall omit that, on purpose to exercise the *Learner's Genius*, and proceed.

PROBLEM VI.

To take the *Distance*, upon level Ground, of any *inaccessible Tree*, *Fort*, &c. or *Breadth* of a *River*, by a *Common Square*.

Suppose there is a River, as ABC, whose *Breadth* you want to know. —First, upon the Bank, at A, set up a Stick, AD, which suppose to be 5 Feet, or 60 Inches high; then fixing your *Square* on the Top, at D, look by the Side of it till you see the Edge of the opposite Shore C, and fasten it, as it may not go from that Position. This done, extend a Thread from D, by the other Side of the *Square* till it touch the Ground at E. Then measure the Distance EA, which suppose 15 Inches, (or 1 Foot 3 Inches) and you may find AC (by Reason of similar Triangles) thus.

Dist. EA : Side DA :: Side DA : Dist. AC
 As 15 ——— 60 ——— 60 ——— 240 Inches,
 which, reduc'd to Feet, give 20 for the Breadth of the *River* sought.



NOTE. There are various Ways of taking *Heights* and *Distances*; but the *best* is to take the Angles for *Heights* by a *Quadrant*; and the Angles for *Distances* by a *Semicircle* or *Theodolite*; and calculate by the foregoing *Axioms*. In all *Heights* the *Triangle* stands *upright*; but in *Distances*, it is supposed to lie *flat* or *horizontal*.

I shall here shew the Learner, how to take the *Breadth* of a *River*, or a *small Distance*, without any *Instrument* whatever; which is thus. Standing upon the Bank, bring down the Edge of your *Hat*, till it appears to touch the opposite Side, then steady your Head by laying your Hand under your Chin, and turn yourself towards some *level Ground*, observing where the Edge of your *Hat* glances upon it; for then, the *Distance* from you to that *Place*, is *equal* to the *Breadth* of the *River*, or *Distance* required.

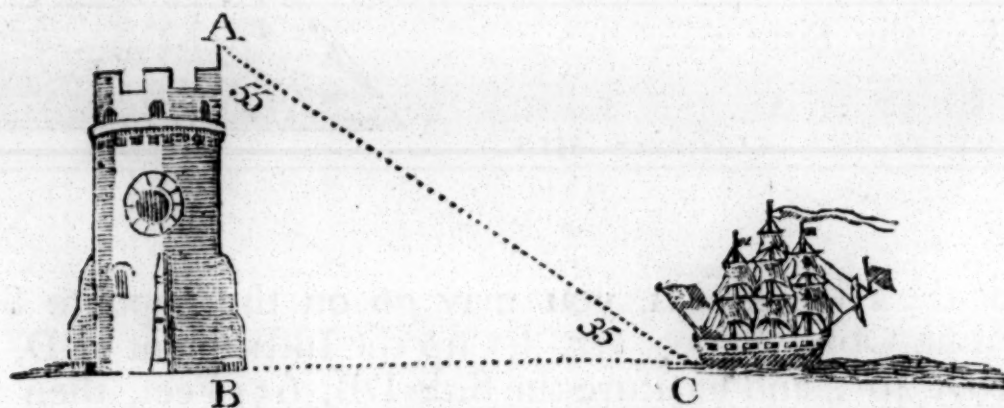
PROBLEM

P R O B L E M VII.

To find, from the Top of a *Fort*, or *Tower*, how far any *Tree*, *Ship*, &c. is from you.

Let A be the Top of a *Tower* or *Castle* standing by the *Sea Side*; and let C be a *Ship* at *Sea*, or lying at *Anchor*, and you would know how far that *Ship* is off the *Castle Wall*.

With your *Quadrant* or *Semicircle*, direct your Sights from the Top of the *Tower* to the Place where the *Ship* is, and take the Angle, which we will suppose to be 55 Degrees. Then the *Castle Wall* being known before to be 143 Feet high, you may easily find the *Distance* of the *Ship* from the *Wall* in this Manner.



(1st.) Find the Side AC.

$$\begin{array}{l} \text{Ang C : Height Wall :: Nat. Rad. : AC} \\ \text{As } 35 \text{ --- } 143 \text{ --- } 61 \text{ --- } 249.2 \end{array}$$

(2d.) Find the Base BC.

$$\begin{array}{l} \text{N. Rad. : AC :: Ang. A : Dist. BC} \\ \text{As } 67 \text{ --- } 249.2 \text{ --- } 55 \text{ --- } 204.56 \end{array}$$

By this Method you may easily discover if a *Fleet* of *Ships*, or one *single Ship*, at *Sea*, makes towards you or not. For having observed from the Top of the *Fort* the Angle from thence to the *Ship*, and noted it down, rest a little Time, and observe again: Then, if the Angle be *bigger* than before, the *Ship* is *departing* from you; but if *less* she is *making towards* you.

PROBLEM VIII.

To take the *Perpendicular Height* of a *Hill* or *Mountain*, and also the *Horizontal Line*, or *Base*, on which it stands.

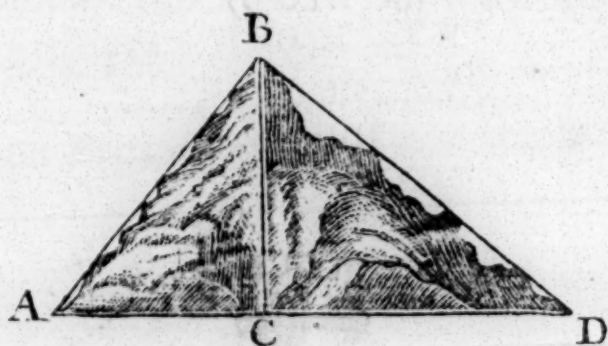
Let ABD be the *Hill*.—First, set up a Mark on the Top at B, equal to the Height of the *Quadrant* or Instrument that is used at the Bottom, from whence you intend to make your Observation. Then by looking through the Sights to B take the Quantity of the Angle at A, which we will suppose to be 50° . Next measure the Hill from A to B, which let be 546 Feet. This being done, you may easily find the *Perpendicular* BC, or Part of the *Base* AC, by Case II. of Right Angles.

For the Perpendicular BC.

$$\begin{array}{l} \text{N. Rad.} : \text{Side AB} :: \text{Ang. A} : \text{Perp. BC} \\ \text{As } 65.2 \text{ — } 546 \text{ — } 50 \text{ — } 418.7 \end{array}$$

For the Side AC.

$$\begin{array}{l} \text{N. Rad.} : \text{Side AB} :: \text{Ang. B} : \text{Side AC} \\ \text{As } 62.2 \text{ — } 546 \text{ — } 40 \text{ — } 351.1 \end{array}$$



Now, as the Hill *descends*, you may go on the opposite Side, and make the like Observations, viz. set up the Instrument at D, and take the Angle D, 40° , and measure the Side DB, 651 Feet, then you may find the Side CD in the same manner you did AC. Thus,

$$\begin{array}{l} \text{Nat. Rad.} : \text{Side BD} :: \text{Ang. B} : \text{Side CD} \\ \text{As } 65.23 \text{ — } 651 \text{ — } 50 \text{ — } 499 \end{array}$$

If to the Part AC = 351.1 Feet, be added the Part CD = 499 Feet, the Sum 850.1 Feet will be the whole Length of the *Horizontal Line* AD requir'd.

The *Perpendicular Height* DC is = 418.7 as above.

PROBLEM

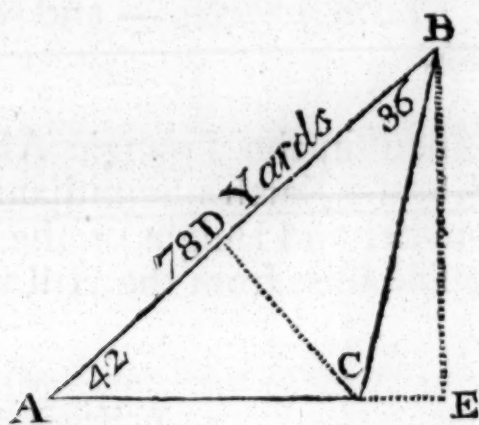
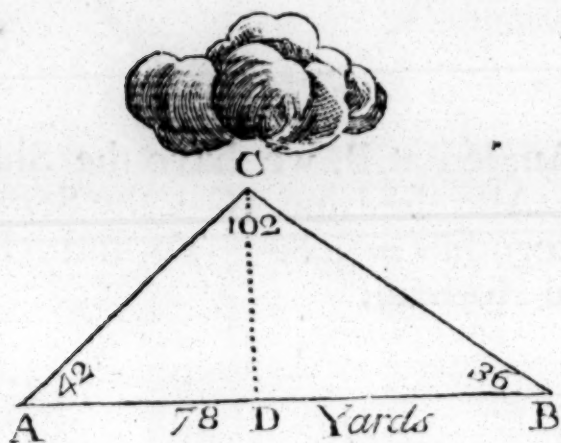
P R O B L E M IX.

To take the *Height* and *Distance* of a *Cloud*.

Suppose it was required to find the Height of the *Cloud* C.

Let a Person standing at A, look through the *Quadrant* to the *Cloud* at C, so will the Thread cut the Angle at A. At the same Time let another Person, making the like Observation at B, take the Angle B. Then measure the *Distance* between the *two Stations*. By this means you will have *one Side* and *all the Angles* of an *Oblique Angle Triangle* given, from whence you may easily obtain the rest, and particularly the *Perpendicular* CD, which will be the *Height* of the *Cloud* required.

EXAMPLE. Suppose the Angle at A, by Observation, be 42° , the Angle B 36° , and the Distance AB 78 Yards: I demand the *Height* of the *Cloud*.



(1st.) Find the Perpendicular BE in Triangle ABE *.

(2d.) Find the Hypotenuse CB in Triangle CBE.

$$\begin{array}{l} \text{N. Rad. : Op. Side AB. :: Ang. A : Perp. BE} \\ \text{As } 62.6 \text{ --- } 78 \text{ --- } 42 \text{ --- } 52.3 \end{array}$$

$$\begin{array}{l} \text{Ang. BCE : Op. Side BE :: N. Rad. : Hyp. BC} \\ \text{As } 78 \text{ --- } 52.3 \text{ --- } 79 \text{ --- } 52.96 \end{array}$$

(3d.) Find the Perpendicular CD in Triangle CDB.

$$\begin{array}{l} \text{N. Rad. : Op. Side BC :: Ang. DBA : Perp. CD} \\ \text{As } 61.1 \text{ --- } 52.96 \text{ --- } 36 \text{ --- } 31.2 \end{array}$$

Answer, 31.2 Yards, the Height required.

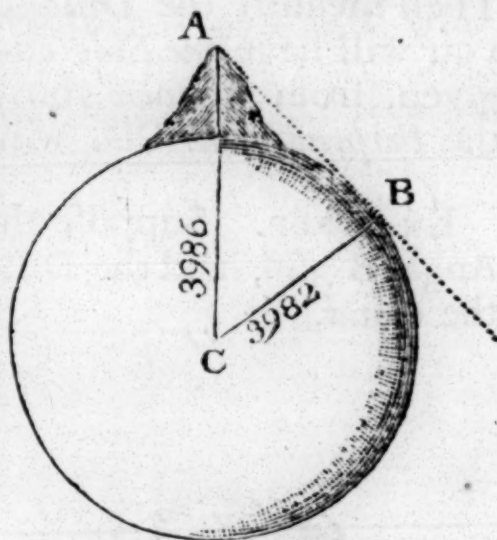
* The Figure on the *Right Hand* is only that on the *Left* set in a different Position, to shew in a more natural or easy Manner, how the Perpendicular falls from the End of the given Side AB, upon the Side AC produced to E.

PROBLEM X.

To find how far a *Hill* of any given *Height* can be seen at *Sea*, or upon *level Ground*.

How far, for Instance, can the *Pike* of *Teneriff* be seen at *Sea*, whose *Height* is about *four Miles*.

The *Circumference* of the *Earth* is sup-
pos'd by *Mathematicians* to be divided into
360 equal Parts, called *Degrees*; and our
countryman, Mr. *Norwood*, has found, by
measuring from the *Tower* of *London* to the
Middle of the *City* of *York*, in the Year
1635, that one of those *Degrees*, upon the
Earth's Surface, contains $69\frac{1}{2}$ Miles; ac-
cording to which Measure, we find the
Earth's Circumference to be 25020 Miles—
its *Diameter* 7964 — and its *Semidiameter*
3982.



Then in the Triangle ABC, Right Angled at B, we have the Side
CB = the *Earth's Semidiameter* 3982. Also the Line AC = the Semi-
diameter and Height of the Mountain together = 3986. To find AB,
the Distance from the Hill to the visible Horizon.

$$\begin{array}{r} \text{To Hypoth. AC} \quad 3986 \\ \text{Add Leg BC} \quad 3982 \\ \hline \text{Sum} \quad 7968 \\ \text{Multiply by Differ.} \quad 4 \end{array}$$

Extract the Root $\sqrt{31872.00}$ (178.5 The Distance the
Mountain can be seen.

$$\begin{array}{r} 27 \overline{)218} \\ 189 \end{array}$$

$$\begin{array}{r} 348 \overline{)2972} \\ 2784 \end{array}$$

$$\begin{array}{r} 3565 \overline{)8800} \\ 17825 \end{array}$$

975

This Mountain can be seen 178.5 Miles at Sea.

PROBLE

P R O B L E M X I.

The *Distance* run at Sea, and the *Course*, given; to find the *Difference* of *Latitude* and *Departure* from the *Meridian*.

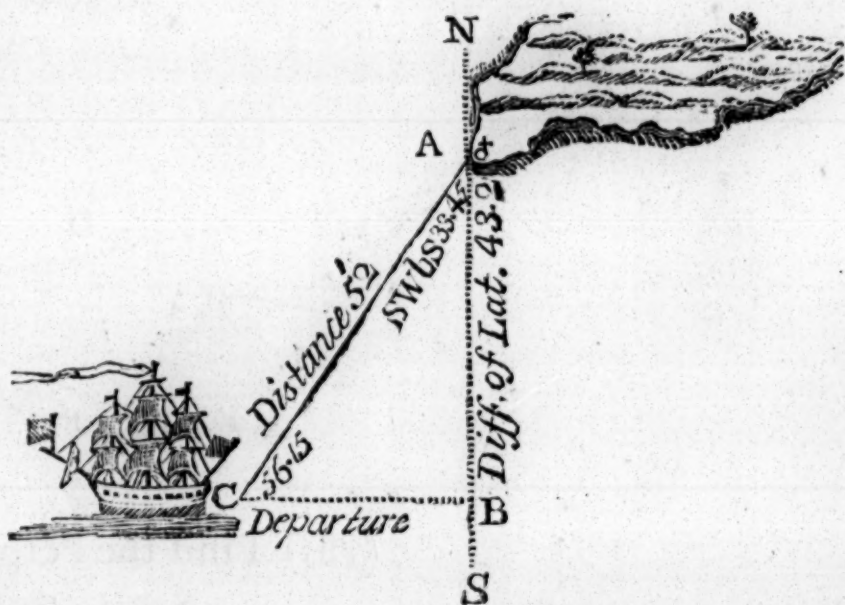
Suppose a *Ship* from A, in the *Latitude* of 50° North, sails away, SW by S. 52 Miles, to C: I demand the *Latitude* she is in, and also her *Departure* from the *Meridian*.

```

33.75
33.75
-----
16875
23625
10125
10125
-----
1139.0625
3
-----
3.417.1875
57.3

```

Nat. Rad. 60.7 is enough.



(1st.) For the Departure.

```

N. Rad. Hyp. AC  ∠ A
As 60.7 — 52 — 33.75
                    52
                    -----
                    6750
                    16875
                    -----
60.7)1755.00(28.9 The Departure
    1214
    -----
    5410
    4856
    -----
    5540
    5463
    -----
    77

```

Latitude departed from	50° North
Difference of Latitude	0 43.2
Latitude the Ship is in	49 16.8

(2d.) For the Diff. of Latitude AB.

```

To Hypoth. AC 52
Add Side BC 28.9
-----
Sum 80.9
Multiply by Differ. 23.1
-----
809
2427
1618
-----
Extract the Root 1868 79(43.2 Differ. of Latitude
16
-----
83)268
249
-----
862)1979
1724
-----
255

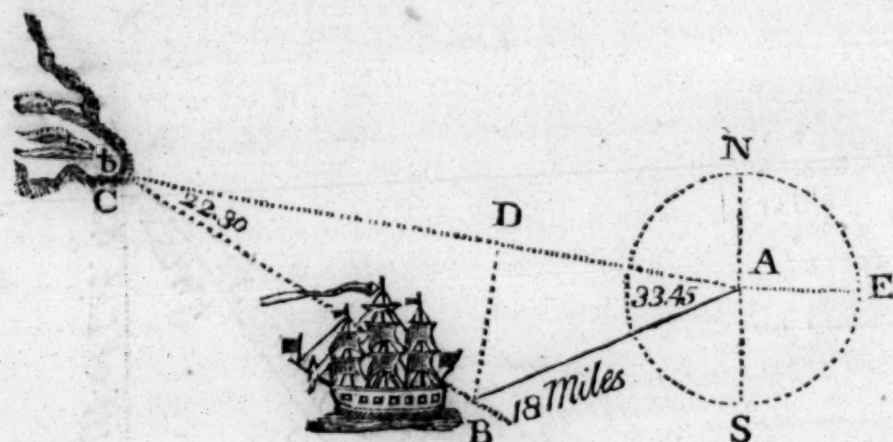
```

NOTE. That in all Cases of *Sailing*, we suppose the *Top* of the Book North, and *Bottom* South; the *Right Hand* East, *Left Hand* West. The *Distance* run is the *Hypothenufe*; the *Difference* of *Latitude* is the *Perpendicular*; the *Departure* the *Base*. The *Angle* at the *Perpendicular* is the *Course*, and the *other* its *Complement*.

PROBLEM XII.

To take the *Distance* of any *Cape, Fort, or Island*, from a *Ship at Sea*.

Sailing W. S. W. I saw, at some Distance, a Point of Land, which I *set*, and find it bears from me W. by N. and having sailed 6 Leagues further, I find it then bears from me N. W. by W. I would know how far this Land is from me.



(1st.) Find the Perpend. BD in Triangle ABD.

$$\begin{array}{r}
 33.75 \\
 33.75 \\
 \hline
 16875 \\
 23625 \\
 10125 \\
 10125 \\
 \hline
 1139.0625 \\
 3 \\
 \hline
 3.417.1875 \\
 57.3
 \end{array}$$

Nat. Rad. 60.7 is enough.

N. Rad.	Op. Side AB	∠ A
As 60.7	18	33.75
		18
		<hr/>
		27000
		3375
		<hr/>
		60.7)607.50(10 Perpend.
		607
		<hr/>
		.50

(2d.) Find the Distance CB in Triangle BCD.

$$\begin{array}{r}
 22.5 \\
 22.5 \\
 \hline
 1125 \\
 450 \\
 450 \\
 \hline
 506.25 \\
 3 \\
 \hline
 1.518.75 \\
 57.3
 \end{array}$$

N. Rad. 58.8 is enough.

∠ C	Op. Side BD	Nat. Rad.
As 22.5	10	58.8
		10
		<hr/>
		22.5)5880(26.13
		450
		<hr/>
		1380
		1350
		<hr/>
		300
		225
		<hr/>
		750
		675
		<hr/>
		75

Answer, 26.13 Miles, the Distance required.

PROBLEM

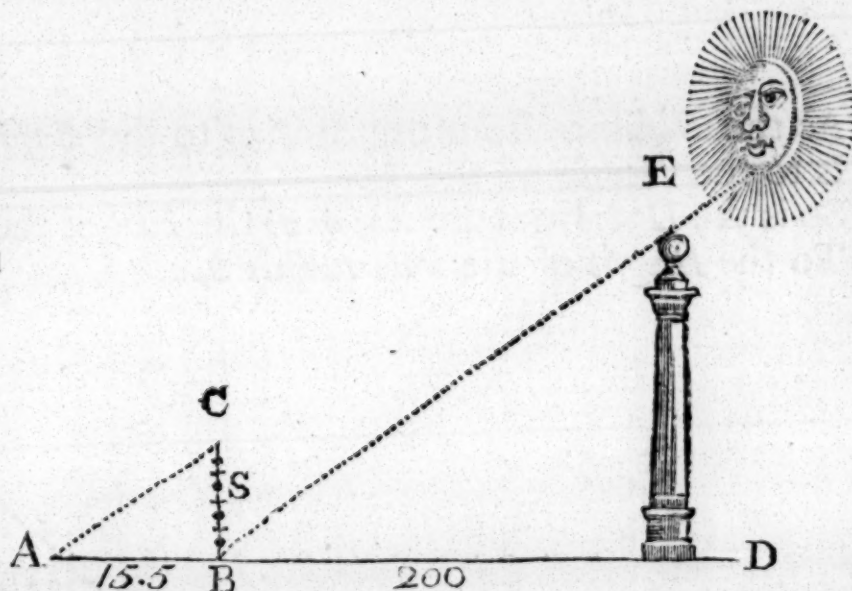
P R O B L E M XIII.

To take the Height of a *Tree, Fort, Obelisk, Pyramid, or any Object*, by a *common Stick* only, when the *Sun or Moon* shines upon it.

Take a Stick of any Length, suppose 8 Feet; set it upright upon the Ground, as at CB in the Figure below. Mark the End of its Shadow at A, and measure its Length from B to A, which suppose to be 15.5 Feet. Then measure the Length of the Shadow of the *Pillar or Obelisk* BD, which suppose to be 200 Feet. This being done, you may easily find the Height: For (by Reason of like Triangles) it will always hold,—as the Length of the Shadow of the Stick AB in the small Triangle, is to its Height CB; so is the Length of the Shadow of the Obelisk BD in the great Triangle, to DE the Height thereof.

Shad. St. Stick. Shad. Ob.
As 15.5 — 8 — 200

15.5)1600.0(103.2
155. . Height
— requir'd
..500
465
—
..350
310
—
..40



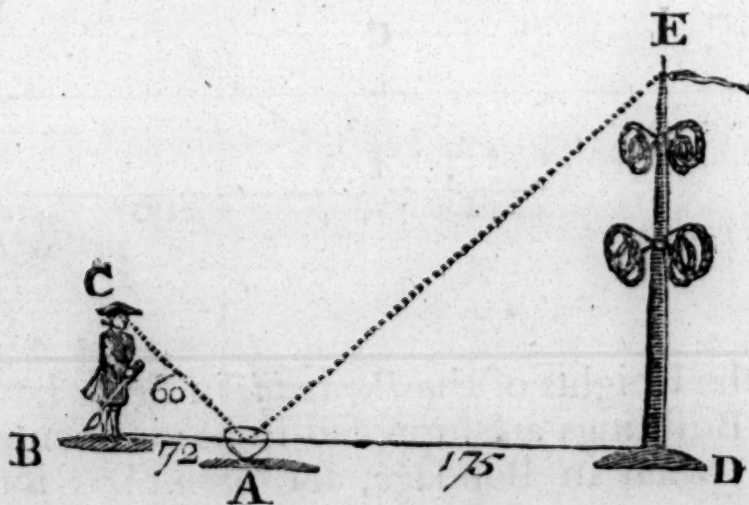
In this *Manner* the Heights of the *Pyramids in Egypt* have been taken. Those stupendous Buildings are supposed to have been erected by the *Children of Israel*, when in Bondage, for *Sepulchers* for the *Egyptian Kings*. They are the greatest Pieces of *Antiquity* now in Existence. There are several *smaller Ones*, but the *largest*, which is justly esteemed one of the *Wonders of the World*, is 500 Feet in *Perpendicular Height*; —700 Feet if measured *obliquely* from the Bottom to the Top;—and its *Base* covers about 11 *Acres of Ground*.

PROBLEM XIV.

To take the *Height* of any *accessible Object* by a *Bason of Water*, or common *Looking Glafs*.

Travelling along the Road, I see a fine *May Pole*, whose *Height* I would gladly know; but having no *Mathematical Instrument* with me, I procure a *Bason of Water*, which I set upon the Ground, at some Distance from the Pole, as at A; then I go backwards, till I see the Top of the Pole in the Middle of the Water, as at B. This done, I measure the Distance from my Station at B to the Bason at A, which suppose 72 Inches; and also measure from the Bason to the Bottom of the Pole at D, and find it 175 Inches. Next I measure the Height of the Eye from the Ground, which suppose 60 Inches. Then say, by the *Rule of Three*,

As the <i>Distance</i> from my <i>Station</i> to the <i>Bason</i> ,	= 72	} Inches.
Is to the <i>Height</i> of the <i>Eye</i> ,	= 60	
So is the <i>Dist.</i> from the <i>Bason</i> to the <i>Foot</i> of the <i>Pole</i> ,	= 175	
To the <i>Height</i> of the <i>Pole</i> requir'd,	= 145.8	



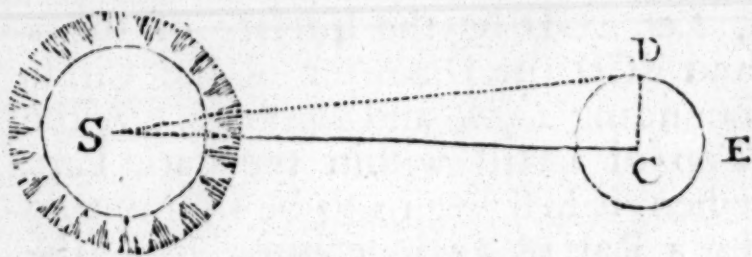
The same Thing may be obtain'd by a *Looking Glafs*, laid truly *Horizontal*, or level on the Ground, by walking back till you can see the *Top* of the *Building*, &c. in the *Middle* of it, as was done by the *Water*.

P R O B L E M X V I I .

To determine the *Distance* of the *Sun*, and all the *Planets*, more accurately than in the last Problem.

The *Horizontal Parallax* of the *Moon* being very difficult, if not impossible, to determine with Accuracy, on Account of the Uncertainty and Mutability of the Horizontal Refractions, which are varying according to the State of the *Atmosphere*: Astronomers have pursued other Methods of doing it. That which seem'd to bid the fairest — was to determine the Parallax of *Venus*, at a Time that, that Planet TRANSITS the *Sun's Disk*. This Method was first propos'd by Dr. Halley, and many accurate Observations were made in distant Parts of the Globe, according to his Proposal. The Result of the several Observations made here and abroad was—that the *Parallax of the Sun* on the Day of the *Transit*, June 6, 1761, was 8'.52. At which Time the Sun was nearly at his greatest Distance from the Earth: Consequently, his Parallax at his *mean Distance* will be something more, viz. 8".65.

Let S represent the *Sun*, E the *Earth*; then in the Triangle SCD Right Angled at C; these are given the Angle at S = 8".65, (the Angle under which the *Semidiameter* of the *Earth* appears (at that Time) at the *Sun*;) and the Side CD = the *Earth's Semidiameter*;—to find the *Distance* SD or SC. Thus,



Angle S : Side CD :: Nat. Rad. : Side SD
As, 0024 ——— 1 ——— 57.3 ——— 23875 Distance of the Sun in Semidiameters of the Earth,

which multiplied by the Earth's Semidiameter gives upwards of 95 Millions of Miles.—The *Sun*, and consequently *all the Planets*, by this Observation of the Transit of *Venus* are found to be further off, than by all former Observations, about $\frac{1}{6}$ Part or something more.

The Table at Page 33 of the View of the Heavens, exhibits the Distances of the *Planets*, according to the *late* and *present Calculations*.

☞ The *Distance* of the nearest *Fix'd Star* is so immense, that it cannot be ascertain'd by this Method.

PROBLEM

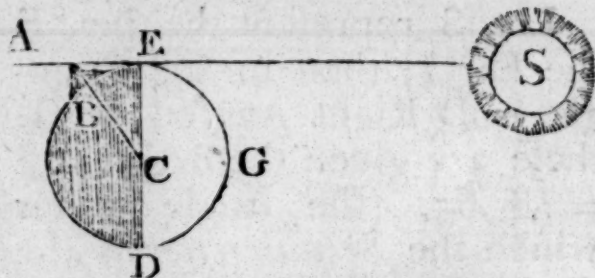
PROBLEM XVIII.

To measure the *Height* of a *Mountain* in the *Moon*.

The *Moon* is suppos'd to be compos'd of *Land* and *Water* as our *Earth* is; consequently there must be some Unevennesses, or Inequalities, of *Hills* and *Vallies* as here. This indeed is confirmed by viewing her thro' a good *Telescope*; for then we find, that the Line, which seperates the Light from the Dark Parts on her Surface is not even or regular, but tooth'd and jagg'd with innumerable Breaks; and even in the Dark Parts, near the Borders of the lucid Surface, there are seen some small Spots enlighten'd by the Sun, which are very visble when the ☾ is three or four Days old, and which can be nothing else but the Tops of *Mountains* or *Rocks*; since it is impossible for the Sun's Rays to fall upon those Parts only, unless they were higher than the Rest of the Surface.

The *Lunar Mountains* are found to be *higher*, in Proportion to the Body of the ☾ than any Hills upon our Globe. The Manner of calculating their Heights is this.

Let EGD be the Surface of the ☾ and ECD the Diameter of the Circle bounding *Light* and *Darkness*. A the Top of a Hill within the dark Part, when it first begins to be illuminated by a Ray of Light coming from the Sun at S. Then observe with a Telescope the Proportion of the Right Line AE, (*i. e.* the Distance of the Point A from the lucid Part) to the Diameter (or Semidiameter) of the ☾ ED, for that being ascertain'd, you have in the Triangle AEC, Right Angled at E, (where the Ray of Light touches the ☾), the two Sides AE and CE, to find the *Hypothenuse* AC, from which subtracting BC = EC, there will remain AB the Height of the *Mountain*.



Ricciolus, on viewing the ☾ when about four Days old, observ'd the Top of a Hill called *Saint Catherine*, near the N. Part of *Mount Taurus*, (see my *Astronomy*) to be illuminated, and that it was then distant from the Surface about $\frac{1}{4}$ of the Moon's *Semidiameter*. Now as the *Semidiameter* of the ☾ EC is about 1088 Miles, the Line AE being $\frac{1}{4}$ of it, must be = 136 Miles. Consequently, if the \square of EC and \square of AE be added together, and then the \square Root of it be extracted, it will give the Line AC, from which subtracting the Moon's *Semidiameter* BC or CE, the Remainder, which is 8 Miles will be the Height of the *Mountain* sought.

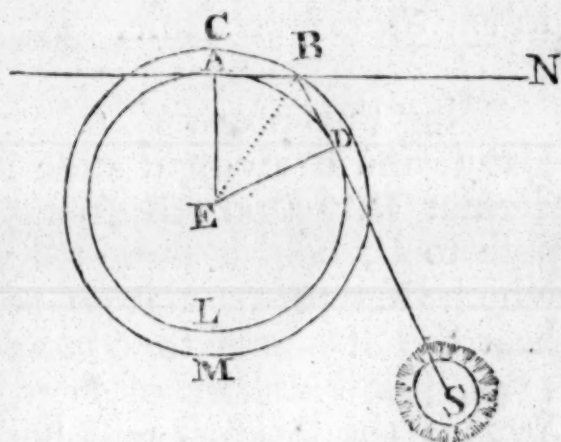
PROBLEM

P R O B L E M X I X .

To measure the *Height* of the *Atmosphere*.

The *Atmosphere* is that Circle of *vaporous Air* surrounding the Earth, which being illuminated by the *Sun's* Rays makes the Brightness and Glory of the *Firmament* we behold, whilst the Sun continues above the *Horizon*. And, after the Sun is gone down, the *Atmosphere*, which is higher than we are, will still continue to be illuminated by those Rays passing by the Earth over our Heads; but this *Brightness* grows *less* and *less* as the Sun descends lower, till he arrives at 18° below the *Horizon*; when all the Parts of the Air above fall out of his Rays, and, consequently, become dark.

To make this plainer; suppose the inner Circle ADL represents the Earth, and the Circle CBM the *Atmosphere*. Suppose a Person standing upon the Earth at A whose *sensible Horizon* is AN. Also let SB be a Ray of Light coming from the *Sun*, touching the Earth at D, which falls upon the distant Part of the Air, in the *Horizon* at B, at which time *Twilight* ceases. This has been found to happen when the Sun is descended 18° below the *western Horizon* in the *Evening*; and also when he is approached within 18° of the *eastern Horizon* in the *Morning*.



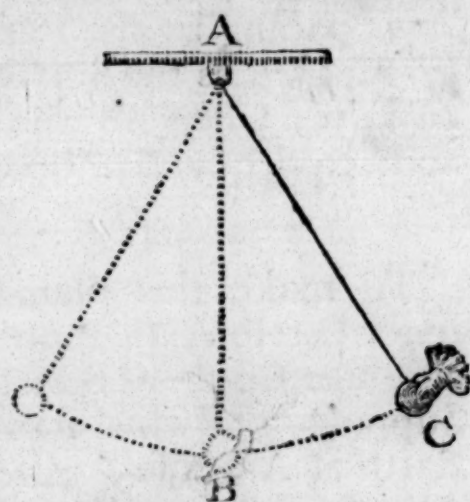
Now as the Arch AD is 18° , we have, by drawing the Line EB, two equal Triangles, from either of which we may find the Height of the *Atmosphere* requir'd.—For in the Triangle ABE, Right Angled at A, we have given the Side AE the *Earth's Semidiameter*, and the Angle AEB = 9° = half the Arch of the *Sun's* Descent below the *Horizon*, and the Angle ABE = 81° , to find the *Hypothenufe* EB, from which if you subtract the *Semidiameter* of the Earth, the Remainder will be the Height of the *Atmosphere*.

Thus, Ang. B	:	Earth's Semid. AE	::	N. Rad.	:	Hypoth. EB
As 81	—	4000	—	81.99	—	4048
						4000 Earth's Semidiameter subtract.
						48 Height of the Atmosphere requir'd.

P R O B L E M XX.

To measure the *Distance* of a *Cloud*, from which issues *Lightnings* and *Thunder*.

Take a small *Ball* of *Lead*, *Ivory*, or any other matter, and affix it to the End of a fine *Thread*. Then measure from the Center of the Ball, along the Thread, exactly 39.2 *Inches*, where make a *Loop*. This done, suspend it by that Loop to the Ceiling of the Room, or to any other Place where it may hang freely, and vibrate backwards and forwards like a *Pendulum*, as in this *Figure*. Now the Property of this little Instrument is, that *each* Vibration, whether it passes through a larger or smaller Space, will be performed in *one Second* of Time.



Being thus prepar'd, take the Ball in your Hand, and drawing it aside from its perpendicular Direction AB, to any Distance, suppose to C, hold it there till you see the flash of *Lightning* pass by, at which Moment let it go, and count the *Number of Vibrations* till you hear the Stroke of the *Thunder*. Then these Vibrations multiplied by 1142, (the Number of Feet, *Sound* uniformly passes through in each *Second*) the Product will be the Height of the *Cloud* in Feet, if it be nearly over the Place where you are; or its Distance from you in any other Situation.

Thus, suppose the *String* is found to make 8 Vibrations between the *Lightning* and the *Thunder*; then $8 \times 1142 = 9136$ Feet, which, divided by 5280 (the Feet in 1 Mile) gives $1\frac{3}{4}$ Mile nearly; and so far is that alarming Tempest from you.

In this Manner you may continue to measure the Distance of the *Cloud* all the Time it passes from your *Zenith* to the *Horizon*, and by that Means be acquainted with the *Danger* it seems to threaten the *Neighbourhood*, as well as the *Extent* of the visible Hemisphere of Clouds.

The *Distance* also of a *Ship* at *Sea*, or a *Fort*, may be estimated in the same Manner, by counting the Vibrations from the *Flash of the Powder* to the *Report of the Gun*.

PROBLEM

PROBLEM XXII.

To calculate the *Diameter* of the *Earth's Shadow* at the *Distance* of the *Moon*; and also, the *Diameter* of the *Moon's Shadow* at the *Earth*.

In the following Figure let S represent the *Sun* E, the *Center* of the *Earth*, M the *Moon*, EC the *Cone* of the *Earth's Shadow* (at a mean) = 215 *Semidiameters* of the *Earth*: Then MC will be the *Cone* of the *Earth's Shadow* reaching beyond the *Moon*, whole Length is thus found.

From EC the Cone of the Earth's Shadow	= 215
Subtract EM the Dist. of the Moon in the Earth's Semid.	= 60
Remains MC the Shadow of the Earth beyond the Moon	= 155

Then, by Reason of similar Triangles, it will always hold;

As the Length of the whole Shadow	EC
Is to the Diameter of the Earth	ab
So is the Length of the Shadow beyond the Moon	MC
To the Diameter of the Shadow at the Moon	cd

EC	:	ab	::	MC
As 215	—	7964	—	155
		155		
		39820		
		39820		
		7964		

215)1234420(5741 Miles = cd, the *Diameter* of the *Earth's Shadow* at the *Distance* of the *Moon*.

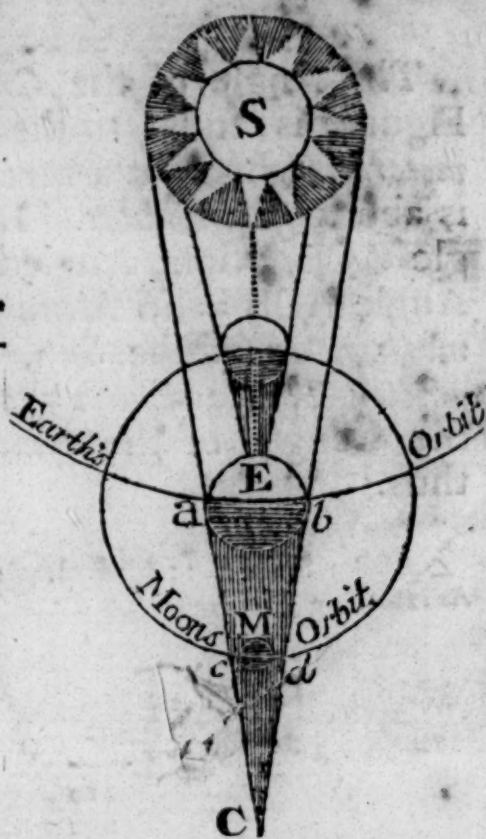
1594
1505
892
860
320
215
105

By this Problem also may be found the *Diameter* of the *Moon's Shadow* at the *Surface* of the *Earth*, and, consequently, how much of the *Earth* is involv'd in that *Shadow* in an *Eclipse* of the *Sun*. For the *Length* of the *Moon's Shadow* is found to be about 60 *Semidiameters* of the *Earth*, which is nearly the

Moon's mean Distance from us; her *Shadow*, in that State, must, therefore, reach as far as the *Center* of the *Earth*.—But as the *Moon* is, sometimes, almost 4 *Semidiameters* of the *Earth* nearer, the *Shadow* must reach 4 *Semidiameters* beyond the *Center* of the *Earth*; and when the *Moon* (as she sometimes is) is 4 *Semidiameters* further from us, the *Shadow* will then not reach the *Earth* at all. In such Case, the *Sun*, though centrally eclips'd, will not be totally covered by the *Moon*; but an *Annulus*, or Ring of Light, will appear round the Border of that Luminary, as happen'd April 1st, 1764.

• The Method of finding the *Distance* of the *Moon*, in *Semidiameters* of the *Earth*, is shewn at Problem XVI.

PROBLEM



P R O B L E M XXIII.

To calculate the *Beginning, End, and total Duration of an Eclipse.*

Having, from *Astronomical Tables*, obtain'd the *Time of the Middle of the Eclipse*, which suppose to be *December 21 Day, 11 Hours, 49 Minutes*, with the *Latitude of the Moon at that Time*, = 40 Minutes, you may then proceed to find the *Beginning, End, and total Duration*, as follows.

From a Scale of equal Parts, of any Size, take off the *Semidiameter of the Earth's Shadow*, which, at the Distance of the ν (at a Mean) is about 42'; and, setting one Foot in C, describe the inner *shaded Circle*, to express that part of the Cone of the Earth's Shadow cut off at that Place where the ν passes through in that *Eclipse*.—With the *Sum of the Semidiameter of the ν = 16', and Earth's Shadow = 42', (which together = 58')* taken from the same equal Parts describe the outer Circle.—Draw the Line FL through the Center, to represent the *Ecliptic*, or Path of the *Earth's Shadow*; cross it, at Right Angles, with the dotted Line PQ, to express the *Poles of the Ecliptic*.—Then with a *Line of Chords*, or a *Protractor*, set off $5\frac{1}{2}^\circ$ from P, upon the outer Circle, towards the *Right Hand*, because the *Latitude of the ν is North ascending*, to express the Angle of the *Moon's Path* with the *Ecliptic*, and draw the Line OR.—Take the *Latitude of the ν = 40'*, from the same Scale of equal Parts, and set it from C to M upon the Line CO.—Then draw a Line through M, at Right Angles to CO, and that Line will represent the *Path of the ν during the Eclipse*.—Next, with the *Semidiameter of the ν = 16'*, taken from the equal Parts, describe, on the three Points B, M, and E, severally, the three *little Circles*; so will the Circle at B represent the ν at the *Beginning*, that at M the *Middle*, and that at E, the *End* of the Eclipse.

Now, from the Center C, draw two Lines to B and E; then in the *Right Angled Triangle CMB*, Right Angled at M, we have given CM the *Latitude of the ν = 40*, and CB = CE the *Sum of the Semidiameters of the Moon and Earth's Shadow = 58*; to find MB = ME, the *Motion of Half Duration of the Eclipse*.

From Square of 58 = 3364
Take Square of 40 = 1600

Extract the Root 1764 (42 Minutes, = the Motion of Half the Duration. And because the Moon passes over 31 of these Minutes (at a mean Rate) in one Hour, we have this Proportion.—If 31 Min. : 1 Hour :: 42 Min. : 1 Hour 21 Min. which subtracted from, and added to, the Middle, will give the Beginning and End.

82)164
164

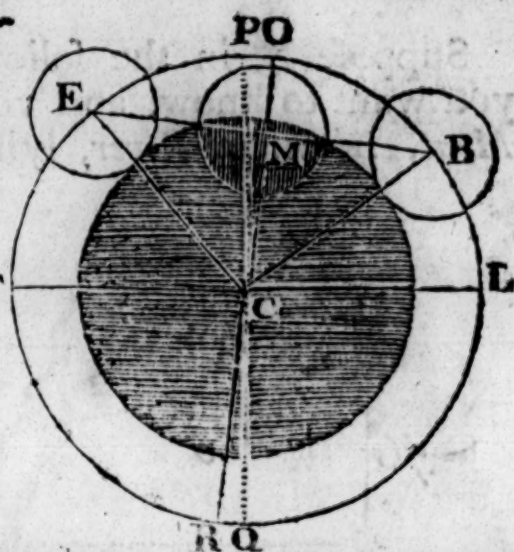
Middle of the Eclipse
Half Duration subtract and add

	D.	H.	M.
Dec.	21	11	49
		1	21

Beginning	21	10	28
End	21	13	10

Total Duration 2 42

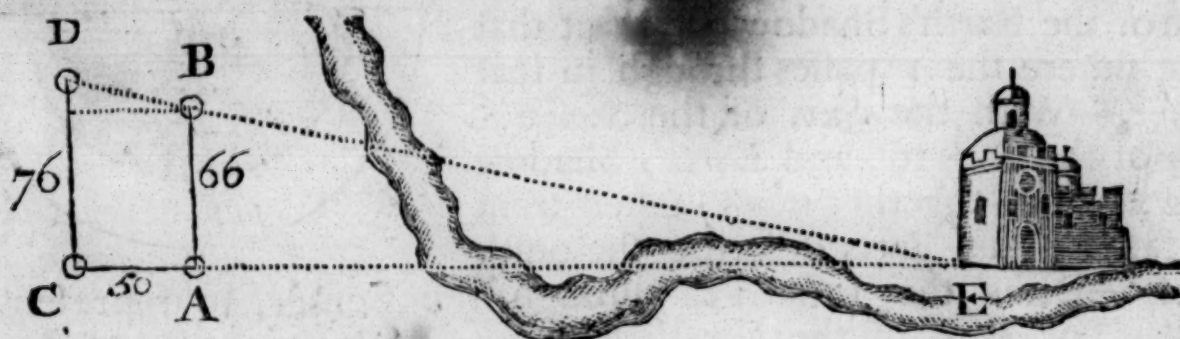
NOTE. The Middle of an Eclipse, with the Latitude of the Moon, may be easily had from my *Astronomy*.



PROBLEM XXIV.

To take the *Distance* of an *inaccessible Object* without the Help of any *Instrument*.

Suppose E, in the following Figure, to be a *Fort*, whose *Distance* you want to know, and you cannot approach it, on Account of some *Moat*, *Ditch*, or *River*, lying between you and the *Object*.



First, at some Distance from the *Ditch* or *River*, set up a *Stick*, as at C; then advance, in a Right Line, towards E, any Number of Yards, suppose 50, and set up another *Stick* at A; next, move, in a Line *perpendicular* to CE, from A to B, any Distance, suppose 66 Yards, and set up another *Stick* at B; then return back to C, where you began, and remove from thence, in a Line *perpendicular* to CE, till you see the *Stick* at B and the *Object* E in a Right Line, and set up another *Stick* in that Place at D, measuring the Distance from C to D, which suppose 76 Yards. Then it will always hold;—

As the Difference between AB and CD	=	10	} Yards.
Is to the Length between A and C	=	50	
So is the Distance CD	=	76	
To the Distance between C and E	=	380	

NOTE. If, in the *third Term*, you had us'd the Distance AB = 66 Yards, you would have obtain'd the Distance from A to E = 330.

PROBLEM

P R O B L E M X X V .

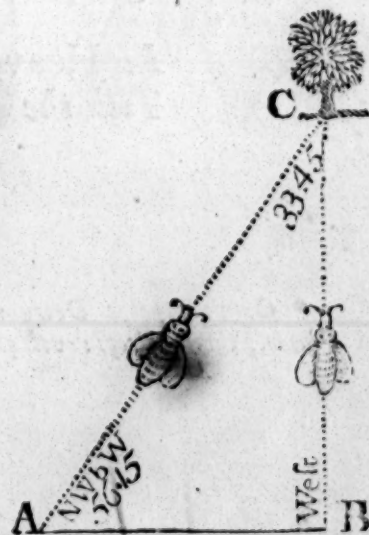
To find, by a *new Method*, where the *Bees* hive in large and extensive Woods, in Order to obtain their *Honey*.

Take a *Plate* or small Piece of *Board*, on which is spread a little *Honey* or *Treacle*, and set it down on a *Rock* or *Stump* of a *Tree* within the *Wood*. This the *Bees* will soon find out if any are near, for it is generally believ'd they smell Things of that Nature at the Distance of a Mile, or further. Whilst these little *Creatures* are feeding, secure two or three of them in a *Box* something convenient. Then let one of them go, observing carefully by a *Pocket Compass*, the Course he takes; for *Bees*, after they rise in the *Air*, fly directly in a straight Line to the *Tree* where their *Hive* is.

Suppose, for Example, the *first Bee* is found to fly directly *West*; then you may be sure the *Tree* is some where in that Line from your present Station. But, in order to know how far, you must make an *Offset*, either *North* or *South*, as large as you can, which in this case we will suppose to be 100 *Rods* or *Perches* (the larger the better) to the *South*. Here you must let go another *Bee*, observing his Course as before, (for this *Bee*, being loaded like the other, will fly *directly* to the *Hive*) which Course we will suppose to be N. W. by W.— $56^{\circ} 15'$ towards the *West*, it only remains now to find where these two Courses or Lines intersect or meet with each other, for there you will find the *Tree* in which the *Honey* is.

This may be easily done.—For in the *Right Angled Triangle* ABC, are given the Right Angle B, the *Course* of the first *Bee*, the Angle at A, the *Course* of the second *Bee*, and the Distance AB; to find BC, or AC, the Distance of the *Tree* from either Station.

$$\begin{array}{l} \angle C : \text{Base} :: \text{N. Rad.} : \text{Dist. AC} \\ \text{As } 33.75 \quad - \quad 100 \quad - \quad 60.7 \quad - \quad 179.8 \text{ Perches.} \end{array}$$



Formerly, they found the *Honey* by surprizing the *Bees*, and following them, one after another, till they found out the *Hive*; but since this *Trigonometrical Method* has been us'd, the Searchers discover that *Booty* in a few Hours, which before requir'd many Days.

CONCLUSION.

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THESE few Problems are sufficient to point out the great Use of this Branch of Learning. The Advantages resulting from it to Society are very great;—almost infinite.—Nothing however posited in the Heavens;—nothing upon the Earth or Seas;—but its Distance and Dimensions may be ascertained by it.—It is no Wonder then, that Pythagoras, a learned Philosopher of Samos, when he had discover'd that famous Proposition (the 47th of the 1st Book of Euclid) which is the Foundation of this Science, offered in Gratitude, sacrifice an He-catomb, i. e. 100 Oxen, to the Gods, inspiring him with such an useful Invention, which he judiciously ascribed the Power of human Abilities to discover.

Thus by one plain Geometrical Figure, having three Sides and three Angles, and assisted by the Rule of Three, you see what amazing Truths may be discover'd. This illustrates not only the old Motto,---*Tria sunt omnia* *---but also proves the Truth of that in the Title-page.

*Cuncta Trigonus habet, satagit quæ docta Mathesis,
Ille aperit clausum quicquid Olympus habet.*

Which may be English'd thus:

*In Heaven the latent Science lay conceal'd,
Till the Triangle came, and Truth reveal'd.*

* *Omnium prope Deorum Potestas triplici Signo ostendatur; ut, Jovis trifidum Fulmen, Neptuni Tridens, Plutonis Canis triceps: vel quod Omnia ternario continentur.*



FINIS.

